Popular Astronomy.

Vol. IX. No. 6. JUNE AND JULY 1901. Whole No. 86.

THE NEW COMET.

E. A. FATH.

FOR POPULAR ASTRONOMY.

To Halls, of Queenstown, Cape Colony, belongs the honor of having discovered the first comet of the century. The discovery was made on the morning of April 24 at which time the comet was near perihelion and was thus quite bright, about first magnitude. It was independently discovered in Australia at Cape Leeuwin and at Arequipa, Peru.

All observations agree in saying that the comet was very bright and could be easily seen by the naked eye for some days after discovery. The head was circular and nebulous, being more than 1′ in diameter, having a well-defined nucleus and a tail variously reported from 2° to 10° in length. Yerkes Observatory reports two tails plainly visible on April 27, while others give it a triple tail.

Thus far the comet has been seen only twice in the northern hemisphere, at the Yerkes and Lick Observatories. A reference to the drawing, which was made from elements computed by Dr. Kreutz of Kiel, the orbit of the Earth being projected on the plane of the comet's orbit, will show why it is seen with so much difficulty in this hemisphere. It had already passed its descending node before discovery and has been south of the ecliptic ever since, being therefore much more favorably situated for observers in the southern hemisphere than in the northern. The newspapers report that at Yerkes the comet was seen 15° north of the Sun on April 27. This cannot be reconciled with our drawing. One or the other must be erroneous. At present the comet is moving directly away from us and will, no doubt, soon be lost to view. It has been diligently looked for here at Northfield but the search has as yet been fruitless.

The three Cape observations and the elements of the orbit computed from them by Dr. Kreutz as received by telegraph are as follows:

Greenwich M. T.	R. A.			Decl.		
1901.	h	m	s	°	′	″
April 24.712	1	30	4	+3	27	0
May 3.2115	3	40	32.4	−0	33	49
4.2187	3	54	29.2	−0	18	27

ELEMENTS.

$T = 1901$ April 24.22 G. M. T.
$\omega = 202° \ 50'$
$\Omega = 109 \ 57$ } Mean Eq. 1901.0
$i = 131 \ 26$
$q = 0.2446.$

EPHEMERIS.

Greenwich Midn.	R. A.			Decl.		Light.
1901.	h	m	s	°	′	
May 12	5	18	16	+2	43	.011
16	5	44	4	4	8	
20	6	4	12	5	21	
24	6	20	24	+6	23	.003

Light, April 24 = 1

VARIABILITY IN LIGHT OF EROS.

EDWARD C. PICKERING.

The discovery by Dr. Oppolzer that the light of Eros is variable suggests some photometric problems of great interest. If, as seems probable, we assume that the variation is due to the rotation of the planet, we can from measures of its light determine the time of rotation, and the direction in space of the axis of rotation. Owing to the varying position of the observer with regard to the planet, much information can be obtained which is impossible in the case of a variable star.

Four corrections must be applied to the observations. First, for the velocity of light; second, for the distance of the Sun and Earth; third, for phase; and fourth, for the direction of the axis of rotation. If this axis were pointed towards the observer, no variation would be perceptible, while the range in brightness would attain its maximum value when the axis was at right angles to the line of sight. Neither of these conditions can be fulfilled exactly, since the position of the axis is probably nearly fixed, and the inclination of the orbits of Eros and the Earth would make great changes in this angle. Let p represent the complement of the altitude of the Earth above the equator of

Eros, which will be equal to the angle between the axis of Eros, and the direction of Eros as seen from the Earth. Let v denote the angle between the plane passing through the Earth and the axis of Eros, and any other plane passing through the axis of Eros, assumed as an origin. A most important correction depends npon v. The time of all the observations must be corrected by an amount equal to v divided by 360° and multiplied by the period of variation. As a first approximation, we may assume that the axis of Eros is parallel to that of the Earth, and that the plane passing through the vernal equinox is taken as an origin. In that case, v will equal the right ascension of Eros. As stated above, if $p = 0°$ there will be no variation in light, and the range will be zero. If $p = 90°$, the range will attain its maximum value. For intermediate values of p, we may assume that the range will be proportional to cos p. The changes in the range may be used to determine the value of p, and from it the position of the axis of Eros. Equations may be formed in which p and v, or p alone, are the unknown quantities from which we may derive the approximate position of the axis. Besides observations at the present time, it will be necessary to determine the light curve when Eros is in several other portions of the sky, determining the range and also the times of maximum and minimum as accurately as possible. The rapid motion of Eros renders it diffieult to compare the observations of different nights, without using different, and in some cases distant, comparison stars. Fortunately, the change in light is so rapid that consecutive observations of a large part of the light curve can generally be made. The opposition of 1894 would have been particularly favorable for these studies, since the declination changed from + 57° to — 14° in a few months, and would thus have furnished large coefficients for determining the value of p, although, as shown below, the range seems to have been small at that time.

Assuming that the variation in light of Eros is due to its rotation, two explanations may be offered as in the case of variable stars of short period (*Proc. Amer. Acad.*, 1881, XVI, 257). First, that Eros is darker on one side than on the other, as is probably the case with Iapetus, the outer satellite of Saturn, and secondly, that it is elongated, or double, as has been assumed by M. André and others (*Astron. Nach.* 155, 30). In the first case, the successive maxima would always have the same intensity, and would succeed each other at equal intervals which would be equal to the period of revolution. The same would be true for the minima. In the second case, if the two bodies differed in di-

ameter, the successive maxima and minima might have unequal intensities, and if the orbit were elliptical the intervals between them would be alternately long and short. This seems to be the case with Eros, and the first hypothesis seems therefore improbable.

On the other hand, if the variation in light is caused by two similar bodies alternately eclipsing each other, it is difficult to see how more than half the light can be cut off in each case, and the minima more than three-quarters of a magnitude fainter than the maxima. It then becomes necessary to assume that the two bodies are of unequal brightness, that they are elongated, or that we have a single body of the shape of a dumb-bell. Some observers have found the minima two magnitudes fainter than the maxima. To account for this, we should be obliged to assume that one axis of the body was six times as long as that at right angles to it. Observations show that the light of Eros is continually varying, while if the case were that of a simple eclipse, as in the stars of Algol type, we should expect that it would retain its full brightness for a large portion of the time.

If the bodies were of the same size, and the orbit circular, it might be impossible, from the light curve, to distinguish between the two hypotheses. The fourth of the corrections mentioned above, however, furnishes a means of distinguishing between them in any case. If the body is dark on one side, the time of revolution will equal the interval between the successive maxima, and the correction for the position of the observer will be proportional to this quantity. If then the position changes 180°, the correction will be one-half the interval between the successive maxima. In the second case, the time of revolution will be double this, that is, equal to the interval between a given maximum and the next but one, so that the correction for position will now be twice as great as before, and approximately equal to the interval between the successive maxima.

Much material already exists for determining the constants mentioned above. Several of the photographs of Eros taken in 1893, 1894 and 1896, had an exposure of an hour or more. Owing to the motion of Eros, it formed a trail on each of these plates, which in some cases shows distinct variations in brightness. This was noticed when the plates were first examined, but was supposed to be due to changes in the haziness of the air. As this is an easy method of discovering the variability of an asteroid, it is hoped that astronomers engaged in a photographic search for such objects will examine carefully all trails, to detect

any changes in intensity. An examination of forty-one asteroid trails photographed with the Bruce telescope, seven of them on a single plate, failed to show, except in one or two instances, any change beyond that apparently due to varying atmospheric absorption. Generally, more than one asteroid appeared on each plate, and in such cases all showed the same changes in intensity.

The photographs of Eros taken in 1893 and 1894 fail to show any marked variations in light, and it is probable that the range was, at that time, small. The first three photographs were taken on October 28, 30, and 31, 1893, and included the same region, so that Eros could be compared with the same stars on all. On the first photograph it was estimated to be 0.20 magn. fainter, and on the second 0.17 magn. fainter than on the third. The corresponding times, expressed in Julian Days and fractions following Greenwich Mean Noon, are 2,412,765.913, 2,412,767.846, and 2,412,768.890, respectively. The corrections mentioned above for velocity of light, and for the position of the Earth, have not been applied. No conclusions can be drawn from the plates taken on January 1 and 8, 1894. The plate taken on January 30, 1894, shows that the light was nearly constant during the first 30^m of the exposure. The Bruce plate taken on February 5, 1894, shows that the light was nearly constant during the first 12^m of the exposure, diminishing by about 0.4 magn. during the remainder of the exposure. A maximum is therefore indicated at about 2,412,865.622.

The plates taken during 1896 give more conclusive evidence of changes. The plate taken on April 6, showed an increase of light during the first part of the exposure, and indicated a probable but somewhat uncertain maximum at 2, 413,656.890. One plate was taken on June 4, and two on June 5. The first of these images was estimated to be 0.20 magn. fainter, and the third 0.83 magn. fainter than the second. The first also indicated a maximum at about 2,413,715.702. The times of the three plates were 2,413,715.694, 2,413,716.829 and 2,413,716.919. A maximum is indicated by the plate taken on June 29, at about 2,413,740.803. The Bruce plate taken on June 30, shows a probable increase, followed by a very marked decrease, and indicating a maximum at 2,413,741.561.

The photometric measures made in 1898, and described in H. C. O. Circular No. 34 (*Astron. Nach.* 147, 363), furnish an accurate determination of the times of maximum, and of the range for that epoch.

A very large number of photometric measures of Eros have

been made since July, 1900. Observations have been obtained with the 15-inch equatorial on 51 nights, the number of photometric settings each night being, in general, 32, but sometimes more. It has also been observed on 56 nights with the 12-inch horizontal telescope, 32 or more settings being made each night. Some months will be required to reduce these observations completely, owing to delay in adopting magnitudes of the comparison stars. It is hoped, however, to issue shortly another Circular giving the results of a preliminary discussion of these observations, and of those described above.

HARVARD COLLEGE OBSERVATORY, Circular No. 58,
April 24, 1901.

THE SCIENTIFIC VALUE OF PHOTOGRAPHY FOR ASTRONOMICAL INVESTIGATIONS.

GEORGE CLARK, M. A.*

The first application of photography to astronomical research was an event no less epoch-making than was the invention of the telescope itself. It has been likened to the invasion of electricity on the old-time methods of illumination; and indeed just as a few brilliant inventions transformed electric lighting from a scientific curiosity into an eminently practical reality, so have the many and varied improvements in photography rendered that art an indispensable auxiliary in any modern Observatory. I purpose in this paper to indicate briefly the nature of the revolution in astronomical method thus brought about, though it is impossible, within the limits of such a paper as this, to give more than a general, and at best very imperfect, sketch of some of the results so obtained. Indeed, there is one highly important department of our work on which I shall hardly touch at all—that of spectroscopic photography; for any description of this must of necessity be more or less technical, and would require, for anything like adequate treatment, a whole paper to itself.

It is just sixty-one years since Daguerre's great discovery first astonished the world, but at that time there were very few who

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dreamed of the future value of photography in the arts and sciences. It is interesting, however, to recall the almost prophetic words of one whose forward gaze *was* keen enough to foresee something of its possibilities—the famous Scottish scientist, Dr. Thomas Dick, whose works on popular science are still useful and delightful reading, and were, moreover, the means of first starting on his magnificent career that giant among astronomers, Professor Barnard, of Yerkes.

In his "Practical Astronomer," published in 1845, Dr. Dick says: "It is not improbable that this art (*i. e.*, photography), when it approximates to perfection, may enable us to take representations of the sublime objects of the heavens. The Sun affords sufficient light for this purpose, and it is by no means improbable that one may obtain an accurate delineation of the lunar world from the Moon herself. . . . Nor is it impossible that the planets, Venus, Mars, Jupiter and Saturn may be depicted in this way, and objects discovered which cannot be descried by means of the telescope. It might, perhaps, be considered beyond the bounds of probability to expect that very distant nebulæ might thus be fixed, and a delineation of these objects produced which shall be capable of being magnified by microscopes; but we ought to consider that the art is yet in its infancy, that plates of a more delicate nature may yet be prepared, and that other properties of light may yet be discovered which shall facilitate such designs."

Were Dr. Dick alive at the present day, he would be amazed to find how much of his prediction has come true. But not even his so active imagination could have foreseen the marvellous extent to which astronomy at the close of the nineteenth century would be influenced by the light-picturing process developed by Daguerre.

Within a few months of the announcement of Daguerre's discovery the first celestial photographs were obtained. These were daguerreotypes of the Moon, taken by Dr. J. W. Draper, of New York, and though they were not very good, they foreshadowed something of future success. But while the daguerreotype was the only available process, no great progress could be made, and the same is true of the collodion method which succeeded it. Both were inherently unsuitable for astronomical work, and it is surprising that any really useful results were accomplished by their aid.

The invention of gelatine dry plates was, however, a decisive event in the history of astronomical photography. Since then

the uses of the camera as an astronomical instrument have multiplied at a prodigious rate. Its versatility appears unbounded. Hardly any task has found it unready or incapable. "It is the very Ariel of the astronomical Prospero." A volume might be written on its successes; while its comparative failures would scarcely fill a page. Few departments of astronomy remain in which the eye has the advantage over it.

Now the reason for this is not, as is often said, that the photographic plate is more sensitive to light than the human retina. That is a popular fallacy. The fact of the matter is, that the eye, if it is to see at all, must see *at the first glance*, while it is the happy faculty of the sensitive plate that by merely looking long enough it can see anything there is to be seen. The human retina can retain an impression only for a short time—perhaps about one-seventh of a second; consequently there is no cumulative effect of the luminous impression to be obtained by prolonged watching. But the sensitive plate knows no such fatigue. The vibrations of the waves of light throw themselves assiduously and persistently on its surface, and steadily apply themselves to the task of shaking asunder the molecules of silver salts in the gelatine film. Just as the waves of ocean by incessantly beating against a shore will gradually wear away the mightiest cliffs, so the innumerable waves of light incessantly impinging upon a single point of the plate will at length effect the necessary decomposition, and so engrave the image of the star.

Obviously this process will be more complete the longer the exposure which is permitted. Now, in taking celestial photographs of very faint objects, we can give exposures of many minutes and even of many hours. Indeed, it is a common thing to give an exposure lasting throughout the whole of three successive nights —the instrument being covered during the day, and the utmost care being taken to keep it directed night after night to the same part of the sky. Every additional minute of exposure means a greater accumulation of the effect of starlight, and thus may mean the bringing of new intelligence from farther and farther sky-depths. Hence it is that a star altogether too feeble to produce any impression upon the most acute eye, even when it is gazing through a telescope of the greatest power, may yet be competent to leave its record on the plate. Thus photographs of the heavens disclose to us the existence of thousands of stars which could never have been detected except for this cumulative method of observation which is the essential prerogative of photography.

You may ask, then, is it possible to penetrate space to an absolutely limitless distance? Does a plate exposed hour after hour and night after night *continue* constantly receiving new impressions of stars, or does there come a time after which no additional amount of exposure will produce a new image on the plate?

The latter appears to be the true state of the case. Dr. Roberts has found that lengthened exposure need not necessarily mean an increased number of stellar images, and that there are even districts in the sky which are, so far as we can ascertain, absolutely void of stars. We are thus led to the belief that the part of the starry realm visible from our Earth is limited in extent. Notwithstanding the enormous assistance afforded by the photographic method, there is still a *ne plus ultra*, where we are again brought to a standstill, perhaps because of the inadequacy of the space-penetrating powers we at present possess, perhaps because there exists only utter emptiness beyond the universe which we know, or perhaps because the ether which seems to us so all-pervading is confined to our universe as the atmosphere is to the Earth, so that there is no vehicle to convey to our senses any information whatever of worlds beyond our ken.

There is another peculiarity about photographic methods of observation which gives them an importance from quite a distinct point of view. The radiation from a star consists of a number of rays of very varied hues all blended together. These rays may be divided into two great groups—those which are able to effect the eye and excite the sense of vision, and those which are incapable of doing so, and are therefore quite invisible. Now, it happens that the rays with which photography is concerned lie partly in the visual portion, but chiefly in the invisible part of the star's radiation. And this affords another reason why the camera may, and indeed must, largely extend our conceptions of the extent of the universe, for it will grasp and depict light which would be utterly wasted so far as vision is concerned. Even if those chemically acting rays were poured in torrents into our eyes, they could excite no sense of vision, and a star whose light is composed of such rays would be absolutely unseeable even in the most powerful telescope, though perfectly capable of being recorded by a photograph.

This phenomenon comes into play in a very pronounced fashion when we attempt to measure by photography the relative magnitudes of the stars. No visual means have hitherto been

found competent to deal with this problem in its completeness. It is easy enough with the help of certain instruments known as photometers to estimate by eye the relative magnitudes of the brighter stars, but with faint stars estimates so made break down. What is called "photographic irradiation" affords, however, another method of photometry. The particles directly meeting the stellar rays reflect them irregularly all around to other particles in the film, thus widening the area of chemical decomposition, and creating circular images which, with the same exposure, and on the same plate, are found to vary in size with the magnitudes of the star they represent—a brighter star producing, *ceteris paribus*, a larger disc on the plate. Even in this way, however, the faintest stars give rise to dots so small that their differences cannot be reliably measured; but for these lowest ranks of stars a fairly accurate estimate of their comparative brightness may be obtained from the relative lengths of exposure necessary to get their impressions at all.

What I wish particularly to draw attention to, however, is this, that the size of the disc produced by a star on a photographic plate is not always a safe guide to the real brightness of the star as we see it in the sky. Red stars, stronger in visual than in photographic rays, give smaller discs; while bluish stars, feebler in optical and richer in photographic rays, make a larger disc than would be expected from their appearance to the eye. There are many stars in the sky which, though apparently single, are really double, consisting of a large star with a smaller one revolving round it. It has been found that the larger star is generally yellow or red, while its smaller companion is commonly a bluish color. But a photograph of such a binary star often shows a curious reversal, the smaller star giving a larger disc than it primary does, simply because of the preponderance of actinic rays in the light it emits. These cases, nevertheless, are the exception and not the rule, for the eye and the sensitive plate in general agree in estimating the graduations of stellar light, and even the exceptions may, I believe, be put right by the use of orthochromatic plates and color screens.

The Sun from its abundance of light, offered special inducements to the early workers in photography, but the method was applied too late to discover any of the great laws of the solar activity. The principal outcome of solar photographs hitherto has been rather to confirm the results previously obtained by patient eye-observation. At the same time photographs of the solar surface in all its varied moods are of extreme interest and impor-

tance. *Facile princeps* in this branch of astronomical photography is M. Janssen of Meudon, who long ago succeeded in making by far the best photographs of portions of the Sun's surface which have ever yet been made. They show the granulations of the surface and the details of the sun-spots and faculæ with admirable clearness. Janssen has always used the old wet-plate process, which, oddly enough, seems to give the best results in solar work. The instrument he uses is a very crude affair. The lens, five inches in diameter, is placed in a wooden box, which is mounted on an old camera stand on rollers. It is not provided with clockwork. When Janssen wishes to photograph the Sun, he wheels this primitive machine, stained and daubed with nitrate of silver, from a shed on to a platform, elevates it toward the Sun, makes an exposure with a rapidly moving slit, and secures a photograph such as hitherto has not been approached in excellence.

From the importance of a more thorough understanding of the effects of the Sun upon the climate of the Earth, daily photographs of the solar surface are made at a number of observatories, *e. g.*, at Greenwich, Kew, and in India. These have been kept up for a great many years, and probably no single day goes by without photographs of the Sun being made at one or other of these stations. One thing which this repeated and constant photographing of the Sun has proved, is the non-existence of any planet nearer the Sun than Mercury. For if such a planet did exist it would frequently be seen to pass across the Sun's disc; and, indeed, before the days of photography many astronomers fancied they had actually witnessed such transits. But in all the long series of solar photographs which we now possess, no strange thing occurs, unless, perhaps, on one made in India, where a distant bird was caught in its flight, and shows its head and outspread wings clearly projected against the Sun.

Probably the most important contribution to our knowledge of solar physics which photography has made has been during the observation of solar eclipses. A total solar eclipse lasts so short a time—never more than just a few minutes—and there is so much to see and note during its occurrence, that it need not be a matter of surprise that drawings and descriptions of the phenomena observed differ very widely.

As you are no doubt all aware there is one portion of the Sun which the eye of man cannot see by any means whatever, except when the glare of the Sun as we see it is shut out by the occulting Moon. Then and then only is there visible the strange white

sheen of the Corona—that mysterious solar extension of radiant filaments, and beams, and sheets of pearly light, which stream out to a distance numbering many thousands of miles from the solar surfaces, forming an irregular halo with the black globe of the Moon in its apparent centre.

To determine accurately the constitution and form of the corona has long been one of the grand problems of astronomy, and the testimony of the camera in this case is invaluable. For example, at the eclipse of 1878 a great many drawings of the corona were made by eminent astronomers, but a subsequent comparison of these showed that scarcely any two could be supposed to represent the same object, and that none of them at all resembled some photographs which were also made at the same time, and which agreed perfectly with one another. The method of free-hand drawing of the corona received its death-blow at that time, for it showed the utter inability of the average astronomer to sketch or draw what he really saw under the circumstances. Before this time some scientists had thought that the corona was simply an atmospheric effect, but the photographs taken at stations hundreds of miles apart give so precisely the same details of form and structure that no doubt can remain as to its being truly a solar appendage.

In speaking of this eclipse, and of the success attained by photography at that time, it is perhaps well to mention the evil influence which the old method had upon at least one observer. A Mr. Holiday had made observations of the corona from a housetop, and thus describes the result in his report: "As soon as the eclipse was over, I came down from the roof and plunged my head into cold water, for I was violently excited, and before breakfast I had made three drawings from my memoranda." If photography had done nothing more for astronomy than to prevent occurrences of this kind, it would at least deserve lasting respect from a humane point of view.

You are all so familiar with lunar photographs that I need spend but little time on them. Exquisite as many of them are, it is still the case that what is shown by the best lunar photograph has not yet approached what can be seen with a good telescope of very moderate size. Professor Pickering avers that "the best lunar photograph ever taken will not show what can be seen with a six-inch telescope under favorable atmospheric conditions." For general outlines, for completeness of coarser detail, and for purposes of future testimony, the photograph clearly stands unrivalled; but as regards what is really most interesting

upon the Moon—the finer detail and more delicate features where alone, if any lunar change ever occurs, that change will show itself—the photograph does not even hint at their existence. Atmospheric agitations are one cause of this imperfection in lunar photographs. The eye can seize the instant of exquisite definition; the camera must take what comes. Another cause is to be found in the disparity of actinic intensity in the various lunar formations, which is so great, that, in order to get an ideal picture, a different length of exposure should be given to each. What is enough for a plain—to take an example— is too much for the crater rising from it, or for the rampart enclosing it.

In the autumn of 1882, the world was thrilled by the advent of a magnificent comet, which suddenly appeared near the Sun in September, and for the next four or five months delighted every one with the splendour of its display. Attracted by the great brilliancy of the comet, Sir David (then Dr.) Gill, at the Cape of Good Hope, with the aid of a local photographer and his 2½-inch Dallmeyer rapid portrait lens of 11-inch focal length, secured a fine series of photographs of the comet with dry plates, and exposures of from half-hour to an hour. When these reached the northern hemisphere they attracted a great deal of attention, not only on account of the comet itself, but also from the number of stars impressed on the plates.

At this time the Brothers Henry of Paris were making a chart of the stars along the ecliptic by eye observations, for the purpose of finding out new minor planets by their slow daily movement amongst the stars. They had at this time reached the region of the Milky Way, and the marvellous wealth of stars they encountered upon entering the boundaries of that vast zone completely discouraged them. At this juncture Dr. Gill's comet photographs came to their notice. They were struck with the great numbers of stars shown on these pictures; it came to them as a revelation of the power of photography in this direction, and incited them to attempt the use of this wonderful process in making their charts. It was to this simple incident that the active application of stellar photography is due. They began at once the construction of a suitable photographic telescope of thirteen and a half inches diameter; it was soon completed, and the exquisite star pictures produced with it, the photographs of nebulæ, and of some of the planets, were perfect revelations.

One of their first and greatest successes was their famous photograph of the Pleiades. In an exposure of an hour 1421 stars were photographed. But that was not all; round one of the principal stars a nebula was discovered whose existence had not

formerly been suspected, and which even afterwards could only be seen with the most powerful of existing telescopes. A later photograph, with a longer exposure, showed still more; for now it was seen that the whole group of stars is filled with an entangling system of nebulous matter, binding together the different stars of the group with misty wreaths and streams of filmy light, nearly all of which is entirely beyond the keenest vision and the most powerful telescope. This is largely due to the fact that nebular light is much more actinic than usual, and therefore, of course, much more photographic. This photograph is interesting for another reason, and indicates very forcibly the revolution brought about in astronomical methods by the aid of photography. M. Wolf, of the Paris Observatory, a most industrious and skilful worker, had devoted three years of unremitting labor to the formation of an accurate chart of the Pleiades. His catalogue contains only 671 stars, so that the photograph done in one hour (or rather three hours) shows more than twice the number of stars, and represents them, moreover, as M. Wolf does not quite, with absolute accuracy and precision

I have remarked that the object with which MM. Henry were charting the heavens was that they might thus pick out any small minor planets by their movements among and between the stars. Curiously enough the photographic methods which they thus introduced not only did away with the necessity for mapping the stars, but also gave a new means for discovering these asteroids. For while the stationary stars make circular images on the plate, the moving asteroid makes a trail, and is thus identified at a glance whenever the plate is developed. The first discovery of this nature was made in 1892; and since then they have been discovered simply wholesale by this method, a very large number having now been observed. Indeed the supply of goddesses after whom to name these worldlets is running out, and has to be reinforced by apotheosis or invention, so that already, to some extent, as Professor Holden remarks, the asteroidal catalogue "reads like the Christian names on the roll of a girls' school."

While speaking of discoveries made by the aid of photography, I must just mention two others, namely, the discovery of double stars in which the components are so close as to defy separation visually; and the discovery of many stars whose brightness varies, and whose images on the plate are consequently periodically larger and smaller. I ought to say, too, that star photographs are susceptible of very accurate measurement, and are eminently suited in this respect to afford data from which to esti mate the distance of the stars. This is derived from a series of measures, made at different seasons of the year, of the space between the particular star whose distance is required, and another star lying nearly in the same direction. If, therefore, a series of photographs at different seasons be obtained, the measurements made on these will disclose the star's distance, if it be sufficiently near to admit of the application of the process.

I come now to one of the most remarkable results attained in recent times in the domain of astronomical photography, namely, Professor Barnard's pictures of the Milky Way. This stupendous star-cluster is not susceptible of accurate measurement, and hence it is useless to photograph it with a large photographic telescope designed more for accurate representations of star-positions. The aim of the astronomer here is to obtain as correct an idea as possible of the structural peculiarities of the Galaxy. Though so conspicuous an object to the naked eye, the view we thus get of it is not sufficiently tangible, from lack of detail, to enable one to form any more than a crude idea of its coarser features. What we require, therefore, in the study of this wonderful object is something that will increase the penetration of our vision, and at the same time give us a certain amount of accuracy of position with a large field of view, so that we may study its peculiarities of structure in detail, and at the same time closely locate these details with reference to the whole, and thus, by finally putting structure and detail together, form a comprehensive idea, not only of the details themselves, but also of the relation of these features to each other. Now, the long-focus telescope with a very limited field is not capable of dealing with the Milky Way in the manner stated. Its structural details are very large, far larger in general than the field of view of the ordinary photographic telescope, and vastly greater than that of a powerful visual telescope. We want, therefore, a short-focus instrument, one capable of not only taking in a wide part of the sky, but also of giving a brilliant image; or in other words, we require the reduction of the large details to a smaller scale, with a correspondingly great increase of effective light power.

These conditions exist in the large portrait lenses which were needed in the early days of photography to reduce the exposure time by collecting a great quantity of light from the object and which in these days of rapid dry plates are no longer required for portrait work. Taking in some ten or twelve degrees of the sky these lenses are specially suitable for photographing large surfaces, such as are presented by the Milky Way. Those who have seen Milky Way photographs made with an ordinary photographic telescope, or who have tried to make out its complexities with a visual telescope, must be struck with the remarkable beauty of the pictures obtained with these portrait lenses. They show many peculiarities which must materially alter our ideas of the constitution and structure of the Galaxy, and they have brought to light in it a great mass of very diffused nebulous matter, apparently freely mixed with the groundwork of stars, and differing in many respects from the ordinary nebulæ of the sky. It must be in the study by photography of such regions that we shall finally clear away some of the mysteries of the Galaxy, and approach mare nearly to a solution of the universe.

These old-time portrait lenses have made another very important revelation in connexion with comets' tails. The photographs of Swift's comet taken with their aid in 1892 showed the most extraordinary and rapid transformations were taking

place in it. This comet was an insignigcant affair visually, and even in the telescope showed no sign of any untoward features; yet the photographs showed the tail one day whole and next day broken up into streamers, while on another day the plate showed a second comet formed within the first having a system of tails of its own. Other amazing changes of a similar nature have since been observed photographically in several later comets, but all of these marvellous phenomena would have been unknown to astronomers but for these pictures, and these comets would have passed without any unusual notice.

I have mentioned these photographs of the Galaxy and the comets because I should like to emphasize the fact that there is a large part of celestial photography which requires no elaborate or expensive apparatus, and in the delights of which all of you here are perfectly capable of sharing. Each one has, of course, his own particular taste, and many of you may find other departments of your science more fascinating than this; but I am sure that any of you who attempt celestial photography will find yourselves more than repaid, not only by the beauty and novelty of your results, but also by their constant unexpectedness, for it is one of the charms of such work that you really never know just what you will find on your plate.

Even with a small lantern lens of 1½-inch aperature and 5½-inch focal length, wonderful pictures may be got. Moreover, it will photograph in a few minutes what the ordinary quick-acting portrait lens would require several hours to take, and will show faint nebulocities quite beyond the reach of the larger photographic telescopes. The action of these lenses upon celestial objects of large field does not seem to be strictly subject to the ordinary law of the ratio of aperture to focus; it seems to be always quicker with the short focus than the law requires. With the small lens referred to, the Earth-lit portion of the new Moon was readily photographed in a single second, while with a six-inch portrait lens of ratio one-fifth, from twenty to thirty seconds were required to show it well. The brighter cloud forms of the Milky Way were shown in from ten to fifteen minutes' time, while with the large lens upwards of three hours were required; and faint extensions of nebulæ, like that in Orion, have *for the first time* been revealed by these small lenses on plates, and by processes which are in all respeces the same as those you ordinarily employ.

Thus it is no paradox but simple fact to say that the non-possession of elaborate and highly specialised apparatus, far from being an insuperable obstacle, is, in many ways, a positive advantage for celestial photography. There is, therefore, the greater inducement for some of you at least to attempt it; and when you are in the mood for an experiment, or are in want of a new subject, may I hope that you will point your cameras heavenwards, and find a new delight in the portraiture and study of the wonders of the sky?—*The British Journal of Photography*, April 19, 1901.

WHAT ECLIPSES OF THE SUN TEACH US.

DAVID P. TODD.

FOR POPULAR ASTRONOMY.

Will the eclipse really happen? Are you certain of it? These were questions I found many people seriously asking a year ago, when the failure of the meteoric showers in the preceding November had seemed to them, perhaps naturally enough to have discredited the entire range of astronomical knowledge. So pervasive was this quite reasonable scepticism that it even preceded me to Tripoli; and on landing at the *hatoba* there, I was assured with much warmth that there probably would be no eclipse at all, and that I must have made some mistake in coming to Tripoli, for the track certainly would not cover that villaget of the Ottoman empire.

The eclipse did happen, however, almost exactly on time; and astronomers everywhere along the path of totality, from New Orleans and Norfolk in our own country, at the extreme of its track in the west, to Algiers and Tripoli, at its extreme in the Orient, were gratified beyond expectation by the splendid weather and the exceptional harvest of sheaves of new knowledge of the Sun and his mysterious environment of radiant streamers.

Not so confidently by half are they now congregating in the Dutch East Indies, for the coming eclipse of the 18th of May, a return, as eclipse cycles go, of the Caroline Island totality of 1883, and an eclipse which will offer, to the successful observer on the west coast of Sumatra, the longest period of total observation of the Sun ever witnessed during historic time.

A total eclipse lasting six minutes and a half! The average is but three minutes or less, and the eclipse follower is happy if he gets a minute and a half in unsullied skies, as last May—or even 51½ seconds, as in Tripoli of Sahara.

But to what purpose this unprecedented duration, if skies are thick with cloud? This is the disturbing feature of our present expeditions, fast hurrying to Sumatra, Sinkop, and Borneo. We are all as sure that the eclipse will come off, true to magnitude and time, as we are certain of dawn to-morrow: nothing short of cosmic cataclysms can ensue to prevent it.

Following up the precedent of all totalities since the South American eclipse of 1893, the enlightened Netherlands government of Java, under the direction of a committee of the Academy

of Sciences at Batavia, of which Major General Muller is chairman, has published a most timely resumé of all such weather conditions as could be secured; and this has been freely distributed for the guidance of the visiting astronomers from the United States, from Great Britain, from France, the Netherlands and Japan. Of all months in the year, April and May appear to be if not the best, the least bad. Rain there is, in abundance all along this favored equatorial belt of darkness; but whether or not this necessitates noonday cloud—cloud at the critical six and a half minutes—will be better known the day after the eclipse is over. May is the season between monsoons, and so favorable. Probably the best stations are in the interior of Sumatra and Borneo and Celebes, on the higher and less marshy areas; but hither few white men have ever penetrated, and for the most part these precious regions are left sacred to the head-hunting savage.

Our government has already dispatched an expedition en route via Manila and in charge of Professor Skinner, U. S. Navy—a party chosen with much care and equipped under the superintendency of Director Brown of the Naval Observatory. Their instruments are large and cover a wide range of research. The eminent Professor Barnard of the Yerkes Observatory is a member of this staff.

By the liberality of Mr. W. H. Crocker of San Francisco, Professor Campbell, now director of the Lick Observatory who has had much experience in eclipse work, particularly in India, is enabled to send out a well organized party to Sumatra. President Pritchett of the Massachusetts Institute of Technology, and a successful investigator of eclipse problems for the last quarter century, has dispatched an expedition to the belt of totality in charge of Professor Burton, whose ability as an eclipse leader was proved last May in Georgia. Another American expedition, perhaps the only other one, is sent out under the auspices of Amherst college, by the generosity of Mr. Arthur Curtis James, of New York. Possibly also Mr. Lawrence Rotch of the Blue Hill Observatory, accompanied by Professor Upton of Brown University, will visit Padang for observations partly meteorological.

As previously intimated, European expeditions are not wanting, except from Germany. There is said to be a reason why Germany in late years has sent no eclipse expeditions; perhaps it is true. The last one, in the seventies, I think, had the unfortunate experience which sooner or later overtakes the inveterate eclipse

chaser, of clouds during the entire period of the total and partial phases. On its return, the inflexible Bismarck demanded the results when asked to defray the modest budget of expenses. "There are none," said the trembling eclipsers, "the sky, we are sorry to say, was cloudy." "Then pay your own expenses," thundered the Chancellor.

England and France treat their astronomers differently. So significant is the Sun, with its well kept secrets, in the practical economy of one planetary family that these nations, by government patronage, vie with each other and our own in equipping well considered expeditions to the remoter regions of the planet whenever a total eclipse of the sun is astronomically certain, whether meteorologically so or not.

The Astronomer Royal at Greenwich, Professor Turner of Oxford, Professor Newall of Cambridge, Mr. Maunder of the Royal Observatory, also, doubtless, Sir Norman Lockyer, all are either going out themselves for the unprecedented eclipse, or are sending skilled assistants to bring home such valuable records as the facilities of modern photography place within easy reach. Within recent years, Sir Norman Lockyer has had the able and hearty aid, at each eclipse, of the officers and crew of a British man-of-war, following a precedent first established by the American navy in 1860 and 1889. The French navy, too, has repeatedly rendered effective assistance to its eclipse observers, especially Mr. Deslandres of Paris, who is widely known as the discoverer of the rotation of the corona and who is expected in the belt of totality at the critical moment. The marked courtesies and cordial co-operation of the gallant French commandants is not forgotten.

But perhaps the most ambitious of all expeditions for the coming eclipse is that organized by the Netherlands' home government, under the direction of Professor Van de Sande Bakhuyzen, superintendent of the Leyden Observatory. Their station is already chosen at the most favorable spot for a maximum duration of the eclipse, and their plan of attack embodies a more ambitious program than perhaps that of any other party. The chief of the Expedition is Major General Muller, and the astronomer in charge, Professor Wilterdink, who comes out with an exceptionally fine and well-adapted instrumental outfit from Holland. This elaborate party will locate at the south of Padang, on the Sumatran coast, in the environs of Painan.

Why all this? What reason is assignable for this hurrying to and fro of astronomers from world-wide localities, to witness

the simple darkening of the Sun for a few fleeting moments by the grateful intervention of the Moon?

Early in the astronomical dark ages of the 19th century, this question would not have been asked, because everyone seems to have thought the radiant corona nothing whatever but what anyone would expect from the brilliant rays of a glowing body like the Sun glancing along the irregular and serrate periphery of the eclipsing Moon. Sir William Herschel seems to have been almost the sole astronomer to divine the true relation of the solar orb to matters of practical concern for dwellers on the Earth. Just a century ago he wrote, "The influence of this eminent body on the globe we inhabit is so great and so widely diffused that it becomes almost a duty to study the operations which are carried on upon the solar surface." Had astronomers and meteorologists but had the faith to follow up the indications of his keen insight, we might have had practical spectroscopy a half century earlier, and weatherology today on the only sure basis for effective and unerring prediction—a basis that recognizes the Sun and all forms of its radiant energy as regnant over and above everything else in all cosmic concerns of the terrestrial economy.

Within very recent years the adaptation of the original form of spectroscope devised by Fraunhofer, with its prism mounted in front of the object glass, has led to results wholly unpredicted with reference to the Sun and its surroundings. For about thirty years, the existence of the reversing layer enveloping the Sun's photosphere has been accepted on the authority of Professor Young, the American nestor among eclipse observers. But the exact depth of this layer of the Sun was vastly over-estimated; and the last eclipse brings the very important generalization from the careful discussion of "flash spectrum" plates by Professor Frost of the Yerkes Observatory that the thickness of this stratum is only about 500 miles. If it could be seen with the telescope alone, it would be as difficult to discover as an incandescent object the size of a foot-ball at the distance of Yerkes from New York.

Observations after the manner of the old astronomy,—contacts and photographs to afford the mathematical astronomer data for correcting the present increasingly erroneous tables of the Moon's motion—ought not to be neglected, and for a double reason. Not only will the very practical and necessary art of navigation be enhanced thereby, but accurate places of the Moon are now a matter of high importance to the new astronomy as

well. This was brought out with especial prominence during the last eclipse, when Mr. John Evershed, an able young English spectroscopist, sought a station in Algeria, in the edge of the belt of totality, for the purpose of catching the flash spectrum under the most favorable conditions. His confidence in the accuracy of the lunar tables proved to have been misplaced, and while his apparatus was arranged so as to prevent entire loss of effective result the eclipse turned out not to have been total at his station at all. Where is the mathematician, European or American, to pave the way to new tables of the Moon, replacing those of Hansen, now practically half a century old, and already inaccurate enough to land a whole squadron on the beach, if dependence were to be placed in them alone and implicitly?

Of course the corona will be photographed, hundreds, perhaps thousands of times. But the best results are only to be expected from those cameras that have been fitted with a whirling heart-shaped vane, or occulter, somewhat after the manner first invented and successfully employed by Mr. Burckhalter of Oakland, California. This will reduce the actinically bright inner corona to a minimum exposure, at the same time permitting the faint exterior filaments that greatly lengthened the time necessary to impress their rays upon even the most sensitive plate. Should this important type of auxiliary apparatus be as successful as is expected, astronomers may at last have a few coronal streamers at their whole length on a single plate, and with such definiteness that measures of their delicate filaments can be made with that exactness demanded for comparison with mathematical theory. Perhaps, too, the long ecliptic streamers may again be caught by Mrs. Maunder, who has gone with her husband to the Mauritius, to repeat her signal success of the India eclipse of 1898.

The spectrum of the corona is by no means neglected. Photography is of such prime advantage in this work that hundreds of lines are now precisely caught with less trouble than a single line could be measured optically, quarter of a century ago. Professor Wood, of the University of Wisconsin, by a deft and novel combination of spectroscope and polariscope, is expecting to get the Fraunhofer lines with greater clearness than ever before. The chief coronal line comes out at 5303.7, or about thirteen tenth-metres more refrangible than 1474K, by which it has been known since its discovery in 1869. Tests will again be applied, after the principle of Doppler, to ascertain more exactly the rate of rotation of the corona, and to determine whether it revolves more rapidly than the Sun, or lags somewhat behind it. As for consti-

tution, our recent results tend to show rather some things the corona is not, than what it really is. Coronium there is, surely; but what is "coronium?" Professor Nasini of Padua has, he thinks, "run it to Earth," by his discovery in 1898, in gases belched forth from the crater of the Solfatara de Pozzuoli, of a line corresponding in position with the historic coronium line of the corona. Much, however, of the coronal light must originate in something else than coronium, because of the dark lines in its spectrum; and these the plates exposed under the scheme of operations suggested by Professor Wood will doubtless reveal. Calcium, helium and hydrogen fail to show their presence in the corona's spectrum; but the dark lines indicate much solar light, reflected probably from small meteoric particles, possibly the débris of comets tethered by the Sun's overmastering attraction.

Of the polarization of the corona, and the technical details of ascertaining the exact photometric ratio for the light of its different regions we have not space to speak. We must pass to the bolometer, a new instrument in eclipse research first employed at last May's eclipse by Professor Langley's party from the Smithsonian Institute. The rather unexpected result was reached that "the corona is effectively cooler than the bolometer, and appears therefore, neither to reflect much light from the Sun, nor chiefly by virtue of a high temperature to give light of its own, but seems rather to be giving light in a manner not associated with a high temperature, or at least with the preponderance of infra-red rays usual in the spectra of hot bodies." So significant a conclusion is most important if verified at the coming eclipse; and it would seem to tend toward corroboration of the theory that the streamers of the corona partake of the nature of electric discharges, in part at least—a view long maintained by many astronomers and physicists of eminence.

An illustrated paper, which is a reprint of an article appearing in the *Technology Quarterly* for March, 1891, under the title, "The Eclipse Expedition of the Creighton University Expedition to Washington, Georgia," is by William F. Rigge, S. J. It is a worthy paper by an interested astronomer. The cuts which illustrate the paper are well designed and neatly engraved. Mr. Rigge found that the observed times of the contacts at beginning and end of totality were both seven seconds earlier than computed, while the duration of totality was almost exactly as computed.

THE COMPUTATION OF AN EPHEMERIS OF A PLANET OR A COMET.

MARY CLARK TRAYLOR*.

FOR POPULAR ASTRONOMY.

1. *An Ephemeris.*—When a minor planet has been discovered its orbit is computed after it has been observed a few times. By an "observation" of a planet is meant a determination of its apparent (See Art. 5) right ascension and declination at any given time; an approximately correct orbit may be computed from three such observations. When the orbit of the new planet has thus been computed it is customary to publish a prediction of the planet's position in the sky for several days in advance, in order that astronomers may find it readily, and may by further observations accumulate data for a more precise determination of its orbit. Such a prediction, in which are usually given for each date the planet's right ascension, declination and the logarithm of its distance from the Earth, is called an ephemeris. A new comet is similarly treated. Below is given a portion of the ephemeris of the asteroid Eros, for Berlin mean midnight of the indicated dates, which in astronomical reckoning is twelve hours Berlin mean time. (See Art. 8).

	R. A.			Decl.			log Δ.
	h	m	s	°	′	″	
1900, Sept. 1	2	21	34.29	+ 33	38	45.6	9.908790
2	2	22	44.58	34	0	44.5	9.903911
3	2	23	53.71	34	22	47.9	9.899011
4	2	25	1.63	34	44	55.5	9.894091
5	2	26	8.28	+ 35	7	7.4	9.889151

The unit used in measuring the distance is the Earth's mean distance from the Sun. For example, the ephemeris of the asteroid Maria gives for November 22, 1900, $\log \Delta = 0.168138$. $\therefore \Delta = 1.47278$, or the distance of the planet from the Earth was Δ times the distance of the Earth from the Sun. To express this in miles we should multiply 92,800,000 miles, the mean distance of the Earth from the Sun, by Δ.

Our object is to show how such an ephemeris may be computed when the orbit of the planet or comet is known, that is, when the numerical values of certain quantities called the elements of the orbit have been determined. The orbit of a planet will be first considered in Arts. 2-22 inclusive, and later the parabolic orbit of a comet; the infrequent case of a comet traveling in an elliptical orbit of known high eccentricity will not be considered.

* University of Denver, class of 1902.

2. *The Elements of a Planet's Orbit.*—The elements of a planetary orbit are eight in number, and are named below:

1. The epoch.
2. M, the mean anomaly.
3. ω, the distance of the perihelion from the node.
4. Ω, the longitude of the node.
5. i, the inclination.
6. ϕ, the inverse sine of the eccentricity.
7. μ, the mean motion.
8. $\log a$, the logarithm of the semi-major axis.

As an illustration we give the elements of the orbit of the minor planet Eros, taken from the *Berliner Jahrbuch* for 1902. [See note under Art. 8].

Epoch, October 31.5, 1900, Berlin mean time.

M	304	23	59.7	
ω	177	38	41.6	Mean equinox of 1900.0
Ω	303	30	40.4	
i	10	49	38.9	
ϕ	12	52	48.2	
μ	2015″.12740			
$\log a$	0.1638027			

3. *The Epoch, the Mean Anomaly and the Mean Motion.*—The epoch is the time for which the value of M is given. In order to explain the mean anomaly and the mean motion, consider a circle circumscribed about the elliptic orbit, tangent to it at the extremities of the major-axis. When the planet is at its perihelion, let a fictitious planet start at the same point and move with a uniform velocity around the circle, reaching the common starting point at the same time as the real planet. The length of the arc which the fictitious planet has described since both planets last left perihelion is the value of M at any particular time. The mean motion is the increase of M in a mean solar day, and is usually expressed in seconds of arc. If the planet made one revolution around the Sun in exactly 1440 mean solar days, the value of μ would be $\frac{360^\circ}{1440}$ which equals 900″. If the same planet were at perihelion on September 1, at Berlin mean noon, M would be 0° at that instant. Just 27 days later M would be $0^\circ + (27 \times 900'')$, wnich equals 24300″ or 6° 45′ 0″. If M is given for any particular time and μ is known, the value of M at any desired time may be thus quickly computed.

4. *The Quantities* Ω, i *and* ω. The plane of the orbit, extended to the celestial sphere, cuts upon it a great circle, which intersects the ecliptic at two opposite points called the nodes of

the orbit. When the planet passes from the *south* side of the ecliptic to the *north* side, a line from the Sun through the planet, prolonged, strikes the celestial sphere at the *ascending node*. The distance measured on the ecliptic from the vernal equinox eastward to the ascending node is the longitude of the node and is denoted by ☊, which is always positive and less than 360°. For example, if the ascending node lies half way between the winter solstice and vernal equinox, the value of ☊ will be 315°.

Let the radius vector of the planet (the right line from the Sun to the planet) be prolonged through the latter to the celestial sphere, and as the planet moves let the extremity of its radius vector trace a short arc from the ascending node. The angle made by this arc with an arc of the ecliptic measured eastward from the planet's node is the inclination (i) of the plane of the planet's orbit to the plane of the ecliptic.

In Fig. (1) BOB′ is a small portion of the ecliptic and OA a portion of the great circle traced on the celestial sphere by the

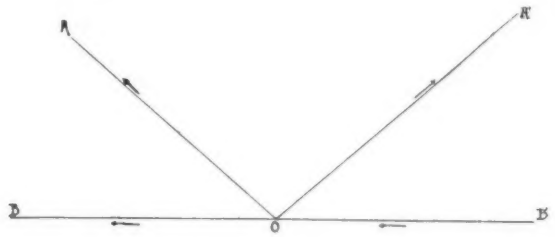


FIG. 1.

plane of the orbit, O being the ascending node. The arrows denote the always eastward directions of the motions of both the Earth and the planet. BOA is i, the inclination. The largest known inclination of a planetary orbit is less than 35°; the inclination of a comet's orbit is reckoned in the same way, but may be greater than 90°. If OA′ be a portion of the trace of the plane of a comet's orbit on the celestial sphere, O being the ascending node and the arrows indicating the direction of the comet's motion, then BOA′ is i. B′OA′ is not the inclination, because B′O is not drawn eastward from O. When the inclination of a comet's orbit is greater than 90° the comet is evidently moving westward. The position on the ecliptic of the ascending node of the orbit is determined by ☊. If i is also known, as explained above, the position of the orbit plane with reference to that of the ecliptic is completely determined.

Let the major axis of the orbit be prolonged through the perihelion to the celestial sphere, meeting it at a point P. The length of an arc of a great circle measured from the ascending node eastward to P is called the distance of the perihelion from the node, and is designated by ω; it may have any positive value less than 360°. The expression "longitude of the perihelion" is applied to the sum of ω and Ω, which is denoted by π.

5. *The Reference Planes.* In Art. 2, in the case of the orbit of Eros the elements ω, Ω and i are said to be referred to the "mean equinox of 1900.0." The year, in our common reckoning, begins in different places at different instants of absolute time; at the moment when any particular year begins at Greenwich it has not yet begun at Washington. In order to secure definiteness and uniformity 1900.0, for example has been defined as the instant at the beginning of the year 1900 when the right ascension of the mean sun is $18^h\ 40^m$. The mean (vernal) equinox is one of the two points of intersection of the mean ecliptic and mean equator. For a further elucidation of this matter we quote from Chauvenet's *Spherical and Practical Astronomy*, Vol. I, Arts. 361-363.

"The variations of astronomical elements are usually divided into *secular* and *periodic.*

Secular variations are very slow changes, which proceed through ages (secula), so that for a number of years, or even centuries in some cases, they are nearly proportional to the time.

Periodic variations are relatively quick changes, which oscillate between their extreme values in so short a period that they cannot be regarded as proportional to the time except for very small intervals.

The *true position* of a celestial body, or of a celestial plane, at a given time is that which it actually has at that time; its *mean position* is that which it would have at that time if it were freed from its periodic variations.

The plane of the ecliptic, or of the Earth's orbit, is a slowly moving plane. Its position at any epoch, as the beginning of the year 1800, can be adopted as a fixed plane, to which its position at any other time may be referred.

The plane of the equator is also a moving plane. Its inclination to the fixed plane and the direction of the line in which it intersects that plane are constantly changing, thus causing variations in the obliquity of the ecliptic and in the positions of the equinoctial points.

The latitudes and declinations of stars are therefore subject to

variations which do not arise from the motions of the stars, but from the shifting of the vernal equinox, which is their common point of reference, or origin, from which both are reckoned.

Under the head of *precession* are considered those parts of these variations which are *secular*, namely, those which arise from the motions of the *mean ecliptic* and the *mean equator.*

Under the head of *nutation* are embraced those parts of these variations which are *periodic*, and result from the difference between the motions of the true ecliptic and equator and those of the mean ecliptic and equator."

6. *The Quantities log a and* ϕ. The size and shape of the ellipse are determined by its semi-major axis and eccentricity, the latter being equal to the quotient of the distance between the foci divided by the major axis. In assigning a numerical value to the semi-major axis a, of a planet's orbit, the semi-major axis of the Earth's orbit is taken as the unit. Because in practice the computer prefers to use the logarithm of a rather than a itself, log a is given as one of the elements. Instead of stating as one of the elements the numerical value of the eccentricity e, it is customary to give the angle ϕ, which is the angle between the minor axis and a line joining one of its extremities with one of the foci; its sine $= e$. Considering the right triangle whose vertices are one focus, an extremity of the minor axis, and the center of the ellipse we have the side apposite ϕ equal to ae, and the hypotenuse equal to a. $\therefore \sin \phi = e$. In Art. 2 we see that in the case of Eros $a = 1.458 +$. The semi-major axis of the orbit is therefore equal to that of the Earth multiplied by $1.458 +$ or 92,800,000 times $1.458 +$. This distance, called the *mean distance* of the planet from the Sun, is much less than the average mean distance of asteroids. The angle ϕ is larger for Eros than the average for asteroids; consequently its orbit is more eccentric than is usual for a planet.

7. *Graphical Illustration of a Planet's Orbit.*—In Fig. (2) the small ellipse represents a planet's orbit, P being the perihelion and S the Sun. NVBC is the ecliptic; NEBF is the great circle in which the orbit plane intersects the celestial sphere, V being the vernal equinox and N the ascending node of the orbit. The angle ENV is i; VBCN counted eastward from the vernal equinox is Ω. SP prolonged strikes the celestial sphere at F. The arc NEBF measured from the ascending node eastward is ω. PQ is the major axis.

We have next to derive the formulæ for determining the position of the planet in its orbit on any given date, the elements of its orbit being known.

8. *Computation of M.*—In order to illustrate our problem, the right ascension, declination and distance from the Earth of Eros will be computed for Greenwich mean midnight of December 25, 1900. According to civil reckoning this is the close of Christmas day; according to astronomical reckoning the selected time is December 25.5, because the astronomical day begins twelve hours later than the civil day. As soon as the necessary formulæ are proved, they will be applied to this problem, the computations being given in full.

The value of M for the chosen time is found as follows: From p. 510 of the *American Ephemeris** for 1900, we find that the longitude of Berlin is $0^h\ 53^m\ 34^s.91$ east of Greenwich, which is equal to 0.037210 of a day. Therefore the epoch October 31.5, 1900, Berlin mean time is October 31.462790 Greenwich mean

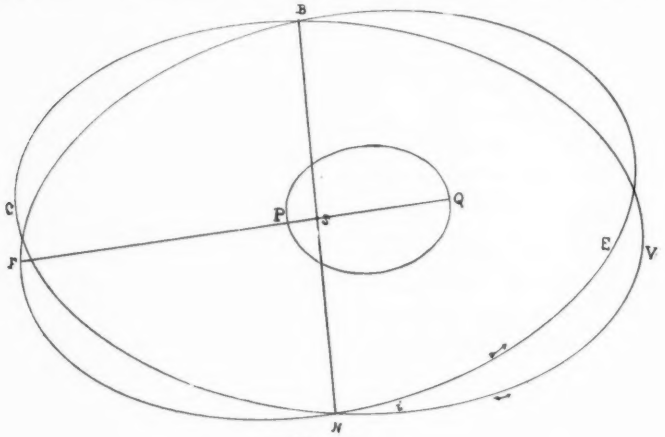


FIG. 2.

time. From this time to Dec. 25.5 Greenwich mean time is 55.037210 days; multiplying μ, which equals 2015″.12740, by 55.037210 and adding to the product the value of M at the epoch we obtain for December 25.5, $M = 335° 12' 26''.7$.

9. *Definitions.*—Fig. (3) represents an ellipse and its circumscribing circle; the Sun is at S, one focus of the planetary orbit APBA′B′, and the planet is at P. A is the perihelion, and SP is the radius vector, r. The angle ASP between the major axis SA and the radius vector is the *true anomaly*, v. The ordinate PM

* The *Berliner Jahrbuch* may be imported through G. E. Stechert, 9 E 16th St., New York City, at a cost of about $3.00. The *American Ephemeris* may be obtained for $1.00 from the Bureau of Equipment, Washington, D. C.

prolonged meets the circle at P′ and the angle ACP′ is the *eccentric anomaly*, E. When the planet is at perihelion $v = 0°$ and $E = 0°$; v and E continually increase, as the planet moves in its orbit, until they become 360° or 0° when the planet again reaches perihelion; they are equal only when the planet is at perihelion or aphelion A′.

10. *Relation between Mean and Eccentric Anomalies.*—The equation of the ellipse is

$$a^2y^2 + b^2x^2 = a^2b^2,$$

while that of the circumscribing circle is

$$x'^2 + y'^2 = a^2.$$

Let x and y be the coördinates of P, and x' and y' the coördinates of P′.

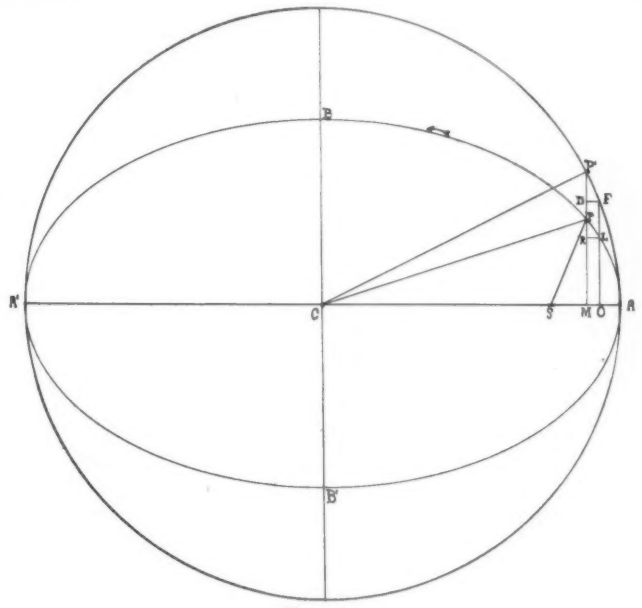


FIG. 3.

Then

$$\frac{y'}{y} = \frac{\frac{a}{b}(a^2 - x'^2)^{\frac{1}{2}}}{(a^2 - x^2)^{\frac{1}{2}}}$$

Since $x = x'$, this becomes

$$\frac{y'}{y} = \frac{a}{b}.$$

Let CA be divided into any number of equal parts and at any two consecutive points of division as O and M let there be ordinates, prolonged to meet the circle at F and P'. Draw FD and LR parallel to CA. Then since any ordinate in the ellipse (as OL) is to the corresponding ordinate in the circle (as OF) as b is to a.

$$\frac{\text{area of rectangle OLRM}}{\text{area of rectangle OFDM}} = \frac{b}{a}.$$

Therefore

$$\frac{\text{sum of rectangles in ellipse}}{\text{sum of rectangles in circle}} = \frac{b}{a}.$$

Hence by the theory of limits

$$\frac{\text{area of ellipse}}{\text{area of circle}} = \frac{b}{a}.$$

Since the area of the circle is πa^2, the area of the ellipse becomes πab.

Since the arc AP' = aE, the area of the sector

$$\text{ACP}' = \frac{a}{2} \cdot a\text{E} = \frac{1}{2} a^2\text{E};$$

then area ACP $= \frac{1}{2} ab \cdot \text{E}.$

Now PM $=$ P'M $\frac{b}{a} = \frac{b}{a} \cdot a \sin \text{E} = b \sin \text{E}$; also CS $= ae$.

$$\therefore \text{Area CSP} = \frac{1}{2} abe \sin \text{E}$$

$$\therefore \text{Area ASP} = \text{ACP} - \text{CSP} = \frac{1}{2} ab\,(\text{E} - e \sin \text{E}) \qquad (1)$$

By one of Kepler's Laws the radius vector describes equal areas in equal times, whence any two areas generated by the radius vector are to each other as the times in which they were generated. Let t denote the number of mean solar days occupied by the planet in moving from A to P, during which the radius vector is generating the area ASP. Let T be the number of mean solar days required for a complete revolution of the planet, during which the entire area of the ellipse is generated by the radius vector.

Then

$$\frac{\text{area ASP}}{\text{area of ellipse}} = \frac{t}{\text{T}}. \qquad (2)$$

Since μ is the mean motion expressed in seconds of arc, $\mu t = M$, the mean anomaly expressed in seconds. Since T is the time of a complete revolution, μT equals 360°, which reduced to seconds equals 1296000″.

$$\therefore \frac{t}{T} = \frac{\mu t}{\mu T} = \frac{M\,(\text{in seconds of arc})}{1296000''} = \frac{M\,(\text{in circular measure})}{2\pi} \quad (3)$$

From (1), (2) and (3) we find

$$\frac{\frac{1}{2}ab\,(E - e\sin E)}{\pi ab} = \frac{M}{2\pi}$$

from which

$$M = E - e\sin E. \quad (4)$$

11. *The Computation of E.*—In our derivation of (4) M and E are both expressed in circular measure. However since the arguments in ordinary logarithmic trigonometric tables are sexagesimal, it is convenient to have M and E in sexagesimal measure. Since a unit of circular measure equals 206264″.8 of sexagesimal measure, to reduce all terms of (4) to sexagesimal measure we multiply them by 206264″.8. The product of e by this number we call e'', which may be read "e in seconds." Hence expressing M and E sexagesimally we write

$$M = E - e''\sin E. \quad (5)$$

In this equation M and e'' are known from the elements of the orbit, while E is to be found. The equation is usually solved by a series of approximations. Let E_1 be an assumed value of E, and $E_1 + x$ the true value; we are to find x.

Let
$$M_1 = E_1 - e''\sin E_1 \quad (6)$$
then
$$\begin{aligned} M_1 + (M - M_1) &= E_1 + x - e''\sin(E_1 + x) \\ &= E_1 + x - e''\sin E_1\cos x - e''\cos E_1\sin x. \end{aligned} \quad (7)$$

When x is small, cos x is nearly equal to unity, and sin x is nearly equal to the circular measure of x; the circular measure of x multiplied by 206264″.8 gives x in seconds of arc. Therefore when x is small and expressed sexagesimally, e'' cos E sin x becomes $e\,x$ cos E. Equation (7) then becomes

$$\begin{aligned} M_1 + (M - M_1) &= E_1 + x - e''\sin E_1 - ex\cos E_1 \\ &= M_1 + (1 - e\cos E_1)\,x, \end{aligned}$$

whence

$$x = \frac{M - M_1}{1 - e\cos E_1} \quad (8.)$$

If (8) were exactly true, the value of E would now be known since $E = E_1 + x$. But (8) is only approximately true, becoming more nearly true as x becomes smaller. Usually $E_1 + \frac{M - M_1}{1 - e \cos E_1}$ is not a sufficiently accurate value of E. Further trial is then necessary, and the process of successive approximation, which is carried on by means of formulæ (7) and (8), may best be explained by an example.

From Art. 8 $M = 335° 12' 26''.7$ for December 25.5, Greenwich mean time; from Art. 2 $\log e = \log \sin \phi = 9.348131$; hence $\log e'' = 4.662556$. Since e'' is not a large angle, it appears from (5) that E and M differ only by a few degrees; sin E must therefore be negative in our example, and E must be smaller than M; as a trial value for the eccentric anomaly we assume $E_1 = 330°$.

E_1	$330°$	$\log e$	9.3481
$\log \sin E_1$	9.69897*n*	$\log \cos E_1$	9.9375
$\log e''$	4.66256	$\log e \cos E_1$	9.2856
$\log e'' \sin E$	4.36153*n*	$\log (1 - e \cos E_1)$	9.9069
$e'' \sin E$	$-22989'' = -6° 23' 9''$	$\log (M - M_1)$	3.6276*n*
M_1	$336° 23' 9''$	$\log$ (approx. x)	3.7207*n*
$M - M_1$	$-1\ 10\ 42 = -4242''$	approx. x	$-5256'' = -1° 27' 36''$

By adding this approximate value of x to E_1 we have as a value of the eccentric anomaly $328° 32' 24''$, which is much more nearly true than $330°$.

Beginning the example afresh we assume that $E_1 = 328° 32' 24''$. Then

$\log \sin E_1$	9.717590*n*	$\log e$	9.348
$\log e''$	4.662556	$\log \cos E_1$	9.931
$\log e'' \sin E_1$	4.380146*n*	$\log e \cos E_1$	9.279
$e'' \sin E_1$	$-23996''.4 = -6° 39' 56''.4$	$\log (1 - e \cos E_1)$	9.908
M_1	$335° 12' 20''.4$	$\log (M - M_1)$	0.799
$M - M_1$	6 .3	$\log$ (approx. x)	0.891
		approx. x	$7''.8$

Hence $E_1 + x = 328° 32' 24'' + 7''.8 = 328° 32' 31''.8$. As x is very small our result must be near the true value of the eccentric anomaly. We now make a final computation, assuming $E_1 = 328° 32' 31''.8$.

$\log \sin E_1$	9.717563*n*	$e'' \sin E_1$	$-23994''.9$ or $-6° 39' 54''.9$
$\log e''$	4.662556	M_1	$335° 12' 26''.7$
$\log e'' \sin E_1$	4.380119*n*	$M - M_1$	0 .0

[NOTE. The letter *n* written after a logarithm indicates that the corresponding natural number is negative. When such a logarithm is added to or subtracted from another which has not the suffix *n*, the sum or difference receives the suffix, because by adding or subtracting two logarithms we indicate the multiplication or division of the corresponding natural numbers, and the product of quotient of a positive and a negative number is negative. If each of two logarithms to be added or subtracted has the suffix *n*, their sum or difference will not have the suffix, because the product or quotient of two negative numbers is positive. It should always be remembered that the suffix does not indicate the sign or the *logarithm*, but that of the *corresponding natural number*.]

Since M_1 now equals M, we have a value of the eccentric anomaly which satisfies (5) and write $E = 328° 32' 31''.8$.

12. *The Radius Vector and True Anomaly.* In Fig. (3) $PM = \frac{b}{a} PM'$.

But $\frac{b}{a} = \sqrt{1-e^2} = \sqrt{1-\sin^2\phi} = \cos\phi$; $PM = r \sin v$ and

$$P'M = a \sin E; \quad \therefore r \sin v = a \cos\phi \sin E. \qquad (9).$$

$$SM = CM - CS.$$

or $$r \cos v = a \cos E - ae = a (\cos E - e) \qquad (10)$$

By Analytic Geometry $r = a - ex$.

$$\therefore r = a (1 - e \cos E) \qquad (11)$$

When E has been computed for any desired date, r and v may be determined from (9) and (10); for by dividing (9) by (10) tan v and v are found. The quadrant in which v lies is determined by the algebraic signs of sin v and cos v, r being always positive. By dividing (9) by sin v or (10) by cos v we may obtain r.

We now derive certain formulæ which may be used as checks on this part of the computation. Subtracting (10) from (11) and substituting $2 \sin^2 \frac{1}{2} v$ for $(1 - \cos v)$, we obtain

$$\sqrt{r} \sin \frac{1}{2} v = \sqrt{a (1+e)} \sin \frac{1}{2} E. \qquad (12)$$

Adding (10) and (11) and substituting $2 \cos^2 \frac{1}{2} v$ for $(1 + \cos v)$, we obtain

$$\sqrt{r} \cos \frac{1}{2} v = \sqrt{a (1-e)} \cos \frac{1}{2} E. \qquad (13)$$

r and v may be obtained from (12) and (13) in the same way as from (9) and (10). When the two polar co-ordinates r and v have been computed, the position of the planet in the plane of its orbit at any given time is completely known.

We now proceed to compute r and v by (9) and (10) for December 25.5, 1900, Greenwich mean time. The values of log a and ϕ used are obtained from Art. 2 and E from Art 11.

log cos E	9.930961	log r sin v	9.870299n
log e	9.348131	log cos v	9.891013
log (cos E − e)	9.799418	log r cos v	9.963221
log a	0.163803	log tan v	9.907078n
log cos ϕ	9.988933	v	321° 4′ 59″.8
log sin E	9.717563n	log r	0.072208

[After v was found log cos v rather than log sin v was used, because the former varies more slowly and the tabular difference in the logarithm table is smaller].

These values of v and $\log r$ may be verified by using the check formulæ (12) and (13).

13. *The Argument of Latitude.* In Fig. (4) the ellipse APN represents the planet's orbit, S being the Sun and A the planet's perihelion; the circle is the trace on the celestial sphere of the plane of the ellipse. D is the ascending node of the orbit; hence DB $= \omega$ (Art. 4); BC $= v$, the true anomaly; DC $= u$, called the argument of latitude, being measured eastward from the node in the plane of the orbit.

$$\therefore u = v + \omega \qquad (14)$$

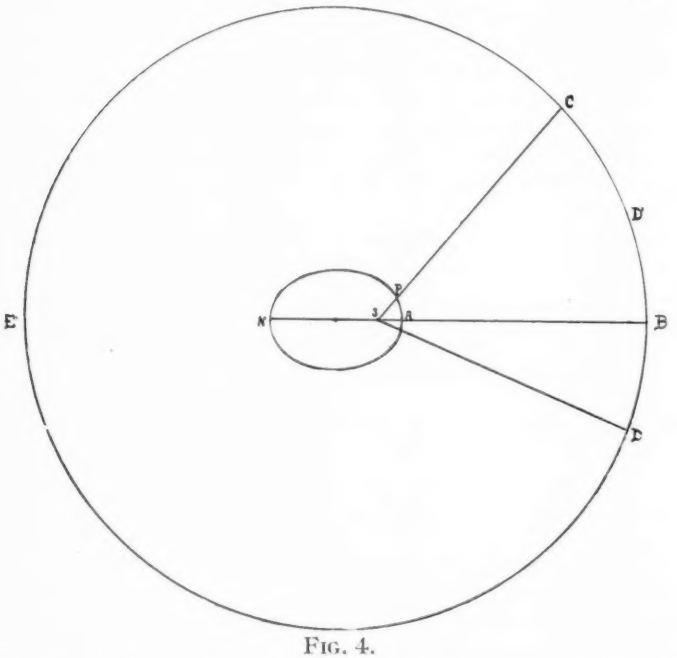


FIG. 4.

In case $v + \omega$ exceeds 360°, as it may, since v and ω may each have any value less than 360°, u is obtained by diminishing the sum by 360°. For example, if the ascending node were at D', ω would be the arc D'CEB and $v + \omega$ would become arc D'CEB + BC which equals 360° + D'C. It is evident that by measuring the small arc D'C, we will arrive at the same point as if we measured the longer arc 360° + D'C eastward from D'.

14. *The Planet's Coördinates Referred to the Ecliptic.* We have now to determine the planet's position with reference to a

system of three rectangular axes, the xy-plane being the plane of the ecliptic, the origin at the Sun, and the z-axis perpendicular to the plane of the ecliptic. In Fig. (5) $XX_0X'X'_0$ represents the plane of the ecliptic; the positive direction of the x-axis, OX_0, lies in the plane of the ecliptic, extending from the Sun to the ascending node; the positive half of the y-axis, OY_0, extends from the Sun to a point in the ecliptic 90° east of the ascending node; the positive half of the z-axis, OZ_0, extends northward from the plane of the ecliptic. P is the position of the planet at any instant; OP is the radius vector r. X_0OP is u, the argument of latitude measured from OX_0 eastward in the plane of the orbit, which passes through the points X_0, P and X'_0. Draw PM perpendicular to the ecliptic and MN perpendicular to OX_0, then join PN, PN will then be perpendicular to OX_0. $PNM = i$, the inclination

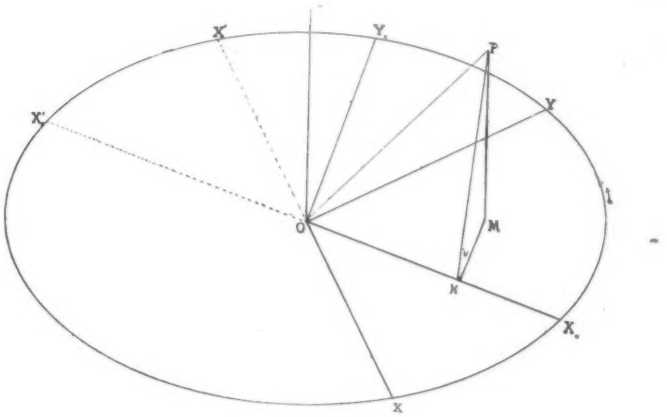


FIG. 5.

of the orbit plane to the plane of the ecliptic. Denote the co-ordinates of P by x_0, y_0 and z_0. Then from the figure

$$x_0 = r \cos u \tag{15}$$
$$y_0 = r \sin u \cos i \tag{16}$$
$$z_0 = r \sin u \sin i \tag{17}$$

Let the present system of co-ordinate axes be turned about OZ_0 as an axis until the positive half of the x-axis passes through the vernal equinox; the positive halves of the new axes are OX, OY and OZ_0. Let x, y and z denote the co-ordinates of Preferred to the new axes. Then

$$PM = z_0 = z. \tag{18}$$

To determine the relation between x, y, x_0 and y_0 we refer to Fig. (6) in which the circle is the ecliptic, and the z-axis is perpendicular to the plane of the paper. From Fig. (5) it is evident that the values of x, y, x_0 and y_0 for P are equal to those of M, its projection on the plane of the ecliptic, and therefore we consider the co-ordinates of M in Fig. (6). MN is perpendicular to OX_0; MR and NT are perpendicular to OX; NQ is perpendicular to MR; the angle $QMN = XOX_0 = \Omega$. $ON = x_0$; $MN = y_0$; $OR = x$; $MR = r$.

$$OR = OT - RT$$
$$x = x_0 \cos \Omega - y_0 \sin \Omega \qquad (19)$$
$$MR = TN + MQ$$
$$y = x_0 \sin \Omega + y_0 \cos \Omega \qquad (20)$$

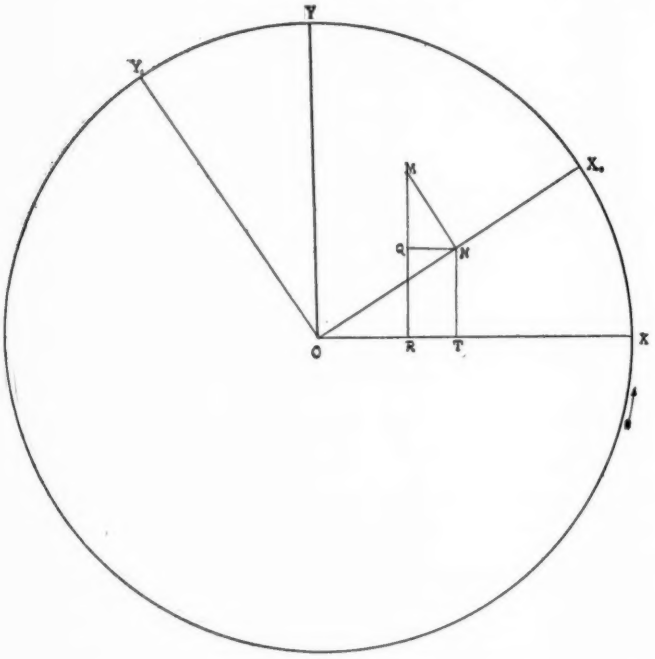


FIG. 6.

Substituting (14), (15), (16) in (17), (18), (19) we obtain

$$x = r\{\cos u \cos \Omega - \sin u \sin \Omega \cos i\} \qquad (21)$$
$$y = r\{\cos u \sin \Omega + \sin u \cos \Omega \cos i\} \qquad (22)$$
$$z = r \sin u \sin i. \qquad (23)$$

15. *The Planet's Coördinates Referred to the Equator.*—Since we wish to find the right ascension and declination of the planet, we must transform equations (21), (22) and (23) from the ecliptic to the equator, by rotating the system of axes about OX through an angle equal to ϵ, the obliquity of the ecliptic. The positive half of the x-axis, running from the Sun to the vernal equinox, is common to both systems; it is perpendicular to the plane of the paper. OY runs, as before, from the Sun

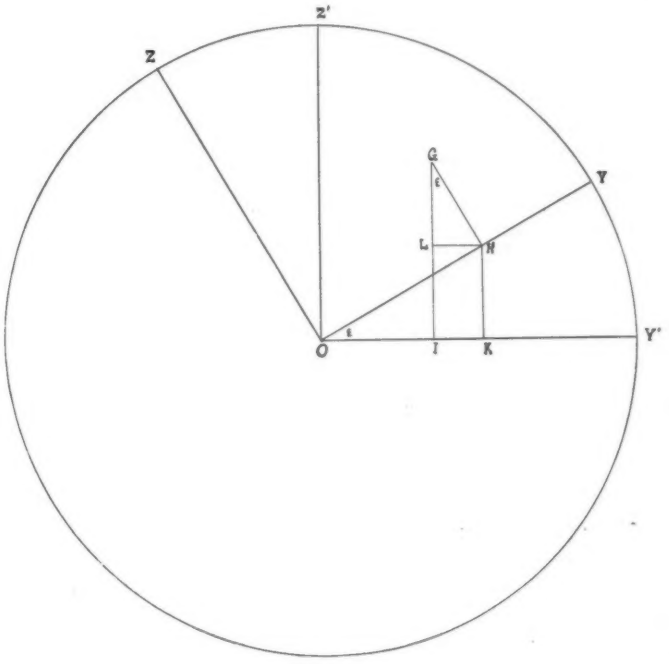


FIG. 7.

to the summer solstice; the circle ZZ'YY' passes through the summer solstice and is perpendicular to both the equator and ecliptic; this circle is the "solstitial colure." OY', the positive half of the new y-axis, strikes the celestial equator at a point whose right ascension is 90°; OZ', the positive direction of the new z-axis strikes the celestial sphere at the north celestial pole. The angle YOY' = ϵ, the obliquity of the ecliptic. Let x', y' and z' be the coördinates, referred to the new axes, of the point P, the position of the planet; G is the projection of P on the yz-plane,

which is the plane of the paper in Fig. (7). GH is drawn perpendicular to OY, GI and HK to OY′, and HL to GI. It is evident that

$$x' = x \qquad (24)$$
$$\text{OI} = \text{OK} - \text{HL}$$
$$y' = y\cos\epsilon - z\sin\epsilon \qquad (25)$$
$$\text{IG} = \text{GL} + \text{HK}$$
$$z' = z\cos\epsilon + y\sin\epsilon \qquad (26)$$

Substituting (21), (22) and (23) in (24), (25) and (26), we have

$$x' = r\{\cos u\cos\Omega - \sin u\sin\Omega\cos i\} \qquad (27)$$
$$y' = r\{\cos u\sin\Omega\cos\epsilon + \sin u\cos\Omega\cos i\cos\epsilon - \sin u\sin i\sin\epsilon\} \qquad (28)$$
$$z' = r\{\sin u\sin i\cos\epsilon + \cos u\sin\Omega\sin\epsilon + \sin u\cos\Omega\cos i\sin\epsilon\} \qquad (29)$$

16. *The Equator Constants.*—In order to simplify equations (27), (28) and (29), there are introduced the auxiliary quantities sin *a*, sin *b*, sin *c*, which are all assumed to be positive, and the angles A, B and C, each of which may have any value less than 360°. These quantities are called the equator constants, and must satisfy the following conditions:

$$\left.\begin{aligned}\sin a\sin\text{A} &= \cos\Omega\\ \sin a\cos\text{A} &= -\sin\Omega\cos i\end{aligned}\right\} \qquad (30)\ (31)$$
$$\left.\begin{aligned}\sin b\sin\text{B} &= \sin\Omega\cos\epsilon\\ \sin b\cos\text{B} &= \cos\Omega\cos i\cos\epsilon - \sin i\sin\epsilon\end{aligned}\right\} \qquad (32)\ (33)$$
$$\left.\begin{aligned}\sin c\sin\text{C} &= \sin\Omega\sin\epsilon\\ \sin c\cos\text{C} &= \cos\Omega\cos i\sin\epsilon + \sin i\cos\epsilon\end{aligned}\right\} \qquad (34)\ (35)$$

The quantities i and Ω are known elements of the planet's orbit and ϵ is the obliquity of the ecliptic, taken from a nautical almanac, such as the *American Ephemeris* or the *Berliner Jahrbuch*. Discussion of equations (30) and (31) will show that it is possible to find values of sin *a*, A, sin *b*, B, etc., which will satisfy the equations (30) to (35). Dividing (30) by (31) we obtain tan A, from which, A can easily be found. The algebraic signs of sin A and cos A, which, since sin *a* is positive are respectively the same as those of cos Ω and $-\sin\Omega\cos i$, determine the quadrant in which A lies. When A is known sin *a* may be found from either (30) or (31).

The solution of equations (32)-(35) may be simplified by introducing further auxiliaries *n* and *N* such that

$$n\sin N = \sin i \qquad (36)$$
$$n\cos N = \cos\Omega\cos i \qquad (37)$$

in which equations, *n* is always positiue and *N* may have any value less than 360°. By substituting (36) and (37) in (33) and also in (35) we derive

$$\sin b\cos\text{B} = n\cos(\text{N} + \epsilon) \qquad (38)$$

and

$$\sin c\cos\text{C} = n\sin(\text{N} + \epsilon). \qquad (39)$$

When n and N have been found from (36) and (37), $\sin b \cos B$ and $\sin c \cos C$ will be computed by (38) and (39) rather than by the more complicated expressions in (33) and (35).

By multiplication we obtain from (33) and (34)

$$\sin b \sin c \cos C \cos B = \sin \Omega \sin \epsilon (\cos \Omega \cos i \cos \epsilon - \sin i \sin \epsilon)$$

and from (32) and (35)

$$\sin b \sin c \sin C \sin B = \sin \Omega \cos \epsilon (\cos \Omega \cos i \sin \epsilon + \sin i \cos \epsilon).$$

By subtraction we have

$$\sin b \sin c \sin (C - B) = -\sin \Omega \sin i.$$

But

$$\sin a \cos A = -\sin \Omega \cos i.$$

Hence

$$\tan i = \frac{\sin b \sin c \sin (C - B)}{\sin a \cos A}. \qquad (40)$$

Equation (40) may be used to check the computation of the equator constants.

Ω	303° 30′ 40″.4	log cos ϵ	9.962555
i	10 49 38 .9	log sin Ω	9.921050n
log (−sin Ω)	9.921050	log sin ϵ	9.599867
log cos i	9.992198		
log sin a sin A	9.742018	log sin b sin B	9.883605n
log cos A	9.918642	log sin B	9.942127n
log sin a cos A	9.913248	log sin b cos B	9.626044
log tan A	9.828770	log tan B	0.257561n
A	33° 59′ 12″.2	B	298° 55′ 35″.8
log sin a	9.994606	log sin b	9.941478
log n sin N	9.273817	log sin c sin C	9.520917n
log cos N	9.975390	log cos C	9.880869
log n cos N	9.734216	log sin c cos C	9.588999
log tan N	9.539601	log tan C	9.931918n
N	19° 6′ 25″.3	C	319° 28′ 21″.7
ϵ	23 27 8 .26	log sin c	9.708130
N + ϵ	42 33 33 .6		
log sin (N + ϵ)	9.830173		
log n	9.758826		
log cos (N + ϵ)	9.867218		

17. *Simplification of Equations (27), (28) and (29).* By substituting (30) and (31) in (27) we obtain

$$x' = r\{\cos u \sin a \sin A + \sin a \sin u \cos A\}$$
$$= r \sin a \sin (A + u). \qquad (41)$$

A similar substitution of (32) and (33) in (28) gives

$$y' = r\{\cos u \sin b \sin B + \sin u \sin b \cos B\}$$
$$= r \sin b \sin (B + u). \qquad (42)$$

In the same manner we obtain from (34), (35) and (29)

$$z' = r\{\sin u \sin c \cos C + \cos u \sin c \sin C\}$$
$$= r \sin c \sin (C + u). \qquad (43)$$

When the values of x', y' and z' are to be computed for several dates, a slight saving of time may be effected by substituting $v + \omega$ for u in (41), (42) and (43) and letting $A + \omega = A'$; $B + \omega = B'$, $C + \omega = C'$. Then

$$x' = r \sin a \sin (A' + v) \quad (44)$$
$$y' = r \sin b \sin (B' + v) \quad (45)$$
$$z' = r \sin c \sin (C' + v) \quad (46)$$

In computing an ephemeris it should be noticed that $\sin a$, $\sin b$, $\sin c$, A', B' and C' are constants computed but once, since they are derived from Ω, i, ϵ and ω. The values of the variables r and v must be found for each date of the ephemeris.

v	321°	4′	59″.8	$\log x'$	9.169957
A′	211	37	53 .8	x'	0.147896
B′	116	34	17 .4		
C′	137	7	3 .3	$\log \sin (B' + v)$	9.989840
$\log r$	0.072208			$\log r \sin b$	0.013686
				$\log y'$	0.003526
$(A' + v)$	172°	42′	53″.6	y'	1.008151
$(B' + v)$	77	39	17 .2		
$(C' + v)$	98	12	3 .1	$\log \sin (C' + v)$	9.995536
				$\log r \sin c$	9.780338
$\log \sin (A' + v)$	9.103143			$\log z'$	9.775874
$\log r \sin a$	0.066814			z'	0.596862

18. *Formulæ for an Extended Ephemeris.* When an ephemeris is to be computed for a large number of dates the formulæ (44)-(46) can be further transformed so as to eliminate r and v, thus rendering their computation for each date unnecessary. Expanding the second members of (44)-(46) we obtain

$$x' = r \sin a \sin A' \cos v + r \sin a \cos A' \sin v \quad (47)$$
$$y' = r \sin b \sin B' \cos v + r \sin b \cos B' \sin v \quad (48)$$
$$z' = r \sin c \sin C' \cos v + r \sin c \cos C' \sin v \quad (49)$$

In these equations substitute for $r \sin v$ the expression $a \cos \phi \sin E$ and for $r \cos v$ its value, $a \cos E - a \sin \phi$. (See Art. 12). Also introduce the auxiliary quantities l', m', n', L′, M′, M′, λ', μ' and ν', whose values may be deduced from the following equations; since their values are the same for each date of the ephemeris, they are to be computed but once:

$$l' \sin L' = a \sin a \sin A' \qquad m' \sin M' = a \sin b \sin B'$$
$$l' \cos L' = a \sin a \cos \phi \cos A' \qquad m' \cos M' = a \sin b \cos \phi \cos B'$$
$$\lambda' = -\sin \phi \, l' \sin L' \qquad \mu' = -\sin \phi \, m' \sin M'$$
$$n' \sin N' = a \sin c \sin C'$$
$$n' \cos N' = a \sin c \cos \phi \cos C'$$
$$\nu' = -\sin \phi \, n' \sin N'$$

Substituting these values in (47)-(49) we obtain

$$x' = l' \sin (E + L') + \lambda'$$
$$y' = m' \sin (E + M') + \mu'$$
$$z' = n' \sin (E + N') + \nu'$$

19. *The Geocentric Coördinates.*—The formulæ of Arts. 17-18 give the coördinates of the planet with reference to axes passing through the Sun. Since we wish to know the position of the planet as seen from the Earth, we choose a new system of axes

which, intersecting at the Earth's center, are parallel respectively to the corresponding axes of Art 18 passing through the Sun. Denote the x-coördinate of the planet with reference to the new axes by ξ, the new y-co-ordinate by η and the z-co-ordinate by ζ. Let the co-ordinates of the Sun with reference to the new axes be X, Y and Z. It is evident that the x-co-ordinates of the planet with reference to the Sun, added to the x-co-ordintae of the Sun with reference to the Earth equals the x-co-ordinate of the planet with reference to the Earth. Similar statements are true of the y and z-co-ordinates. Hence

$$\xi = x' + X \qquad (50)$$
$$\eta = y' + Y \qquad (51)$$
$$\zeta = z' + Z \qquad (52)$$

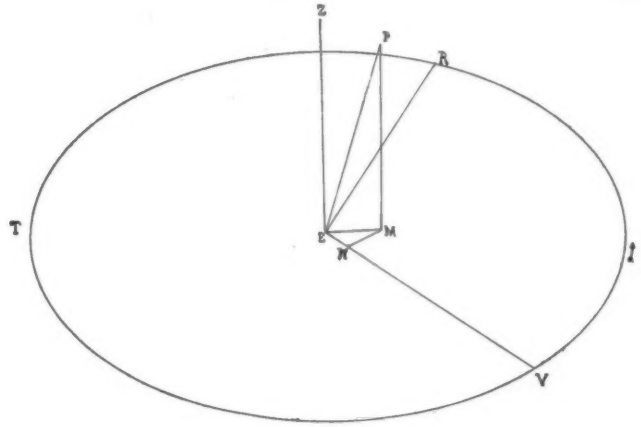


FIG. 8.

X, Y and Z are given in the *American Ephemeris* for each day in the year, at Greenwich noon and midnight. In the *Berliner Jahrbuch* they are similarly given for Berlin noon and midnight.

X	+0.064846	ξ	0.212742
Y	−0.900170	η	0.107981
Z	−0.390510	ζ	0.206352

20. *The Planet's Right Ascension, Declination and Distance from the Earth.* From Arts. 15 and 19 it appears that the positive half of the ξ-axis runs from the Earth's center to the vernal equinox; likewise the positive half of the η-axis runs from the Earth's center to meet the celestial equator at a point whose right ascension is 6^h; the positive half of the ζ-axis strikes the celestial sphere at the north celestial pole. In Fig. (8) VRT is the celestial equator, and V the vernal equinox. R is the point on

the equator which has a right ascension of 6^h. EZ is perpendicular to the plane of the equator. EV is therefore the positive half of the ξ-axis, ER of the η-axis, and EZ of the ζ-axis. The line PM is drawn from the planet P perpendicular to the plane VRT, and MN perpendicular to EV. It is evident that EN $= \xi$, MN $= \eta$ and PM $= \zeta$. EP $= \rho$, the planet's distance from the Earth. The angle PEM $= \delta$, the planet's declination; the angle VEM $= \alpha$, the planet's right ascension. From the right triangles PEM and MEN we at once obtain

$$\rho \cos \delta \cos \alpha = \xi \qquad (53)$$

$$\rho \cos \delta \sin \alpha = \eta \qquad (54)$$

$$\rho \sin \delta = \zeta \qquad (55)$$

ξ, η and ζ being known, α, δ and ρ are found from these equations. For $\tan \alpha = \frac{\eta}{\xi}$; ρ is always accounted positive and $\cos \delta$ is also positive because the absolute value of δ never exceeds 90°; hence the quadrant in which α lies can never be in doubt. Dividing (53) by $\cos \alpha$, or (54) by $\sin \alpha$ we obtain $\rho \cos \delta$. Then $\frac{\rho \sin \delta}{\rho \cos \delta} = \tan \delta$; if $\tan \delta$ is negative δ lies between 0° and $-90°$. ρ may be found from $\frac{\rho \cos \delta}{\cos \delta}$ or $\frac{\rho \sin \delta}{\sin \delta}$.

$\log \rho \cos \delta \sin \alpha$	9.033347	$\log \rho \sin \delta$	9.314608
$\log \cos \alpha$	9.950225	$\log \cos \delta$	9.878717
$\log \rho \cos \delta \cos \alpha$	9.327853	$\log \rho \cos \delta$	9.377628
$\log \tan \alpha$	9.705494	$\log \tan \delta$	9.936980
α	26° 54′ 39″.2	δ	40° 51′ 26″.7
	$= 1^h\ 47^m\ 38^s.61$	$\log \rho$	9.498911

21. *True and Apparent Place.*—The elements of the orbit give the position of the plane of the orbit with reference to the mean equinox and mean ecliptic at the beginning of some year as 1900. Therefore the co-ordinates of the Sun (X, Y, Z) must also be taken with reference to the same positions of the fundamental planes. Then the values of α and δ just obtained will be measured with reference to the mean position of the equator and equinox at the beginning of the year. It is customary, when publishing an ephemeris, to give the *true* place of the planet rather than the mean position. (See Art. 5). Therefore to obtain the *true* values of α and δ, we apply to the *mean* values already computed certain small corrections which are made by the use of data given in the *American Ephemeris* or in the *Berliner Jahrbuch*. If the *American Ephemeris* is used the formulæ are

* Correction to $\alpha = f + f' + \frac{1}{15} g \sin (G + \alpha) \tan \delta$, (56)

Correction to $\delta = g \cos (G + \alpha)$. (57)

The quantities f, f', $\log g$ and G must be taken from the *Ephemeris* for the desired dates. For detailed explanations concerning the origin of these formulæ the reader may consult Chauvenet's *Spherical and Practical Astronomy*, Vol. I, pp. 604ff, or Doolittle's *Practical Astronomy*, pp. 558ff. The formulæ just given are obtained from those near the bottom of page 289 in the *American Ephemeris* (for 1900), by dropping those terms of the formulæ which are due to the proper motion of a fixed star and to aberration. The proper motion terms are dropped because we are dealing with a planet, instead of a fixed star; the aberration terms are dropped because we wish the *true* place, which we proceed to explain. The differences between the mean, true and apparent places of a star are set forth in the following extract from Doolittle's *Practical Astronomy*, (Art. 323, p. 560).

"Suppose the right ascension and declination of a star to be accurately observed with a suitable instrument: the place of the star so determined will be the *apparent place*.

"The apparent direction of a star is affected by aberration, the effect of which will be considered more fully hereafter. If we apply to the apparent right ascension and declination the corrections necessary to free them from the effect of aberration, we have the *true place*.

"If now we apply to this true place the small periodic corrections called nutation, we have as the result the *mean place*."

We now proceed to compute the corrections to α and δ by means of formulæ (56) and (57).

G	11°	5′	G + α	38°	0′
α	26	55	δ	40	51

$\log \frac{1}{15}$	8.8239
$\log \tan \delta$	9.9370
$\log \sin (G + \alpha)$	9.7893
$\log g$	1.4208
$\log \cos (G + \alpha)$	9.8965
$\log g \cos (G + \alpha)$	1.3173
$\log \frac{1}{15} g \sin (G + \alpha) \tan \delta$	9.9710
$\frac{1}{15} g \sin (G + \alpha) \tan \delta$	0ˢ.935
$g \cos (G + \alpha)$	20″.8

0ˢ.935 + 3ˢ.965 = 4ˢ.90, correction to α.

20″.8, correction to δ.

∴ α 1ʰ 47ᵐ 43ˢ.51 ∴ δ 40° 51′ 47″.5

* In the *American Ephemeris* before and after 1900 f and f' are combined in a single term.

22. *Reduction of Elements to a Different Equinox.* It is often the case that the elements are given for the mean equinox of some epoch as 1900.0, while the solar co-ordinates X, Y and Z are given in the Almanac for the mean equinox at the beginning of some other year, as 1909.0. Therefore in order to find the values of ξ, η and ζ from the formulæ of Art. 19 it is necessary to transfer the elements from the mean equinox at the beginning of the one year to the mean equinox at the beginning of the other. The transfer may be accomplished by formulæ such as are given below. For a complete explanation of these formulæ the student is referred to Oppolzer's *Lehrbuch zur Bahnbestimmung*, Vol. I.

$$\Omega_1 = \Omega_0 + l + \cot i\,\pi \sin (\Omega - \Pi)$$

$$\pi_1 = \pi_0 + l - \tan \frac{1}{2} i\,\pi \sin (\Omega - \Pi)$$

$$i_1 = i_0 \quad - \pi \cos (\Omega - \Pi)$$

In these formulæ Ω_0, π_0 and i_0 are the original values as given in the elements; Ω_1, π_1 and i_1 are the required values, and Ω and i are the mean values equal respectively to $\frac{1}{2}(\Omega_0 + \Omega_1)$ and $\frac{1}{2}(i_0 + i_1)$. The values of l, π and Π are taken from the following table which is constructed from Oppolzer's formulæ.

	$\frac{l}{t_1 - t_0}$	$\frac{\pi}{t_1 - t_0}$	Π
	"	"	° ′ ″
1890	50.2437	0.4792	173 22 7
1900	50.2459	0.4792	173 27 35
1910	50.2482	0.4791	173 33 4
1920	50.2505	0.4790	173 38 33

As an example we will transfer the elements of Eros from the mean equinox of 1900.0 to the mean equinox of 1909.0. The values of l, π and Π will be computed for the time 1904.5, which is midway between 1900.0 and 1909.0. As the mean values of Ω and i are unknown, they are in the first approximation assumed to be equal to the original values Ω_0 and i_0. This approximation gives Ω_a, π_a and i_a as approximate values of Ω, π and i. In the second approximation we use for Ω and i, $\frac{1}{2}(\Omega_0 + \Omega_a)$ and $\frac{1}{2}(i_0 + i_a)$. In this particular case the first approximation gave accurate results, but had the time interval been longer, there would have been a difference between the values given by the first and second approximations. In the fol-

lowing computations $\Delta\Omega_{II}$ and $\Delta\pi_{II}$ denote the second terms of the changes of Ω and π respectively:

Ω_0	303°	30′	40″.4
π_0	121	9	22 .0
i_0	10	49	38 .9
l		7	32 .22
π			4 .31
II	173	30	3
$\Omega_0 - II$	130	0	37

$\log \sin (\Omega_0 - II)$	9.8842
$\log \pi$	0.6345
$\log \cos (\Omega_0 - II)$	9.8082n
$\log \pi \sin (\Omega_0 - II)$	0.5187
$\log \pi \cos (\Omega_0 - II)$	0.4427n
$\log \cot i$	0.7183
$\log \tan \frac{1}{2} i$	8.9766
$\log \Delta \Omega_{II}$	1.2370
$\Delta \Omega_{II}$	17″.26
$\log \Delta \pi_{II}$	9.4953
$\Delta \pi_{II}$	0″.31
$\log \Delta i$	0.4427
Δi	2″.77
Ω_a	303° 38′ 29″.9
π_a	121 16 53 .9
i_a	10 49 41 .7

Ω	303° 34′ 35″.2
i	10 49 40 .3
$\Omega - II$	130 4 32 .2
$\log \sin (\Omega - II)$	9.8838
$\log \pi$	0.6345
$\log \cos (\Omega - II)$	9.8088n
$\log \pi \sin (\Omega - II)$	0.5183
$\log \pi \cos (\Omega - II)$	0.4433n
$\log \cot i$	0.7183
$\log \tan \frac{1}{2} i$	8.9766
$\log \Delta \Omega_{II}$	1.2366
$\Delta \Omega_{II}$	17″.24
$\log \Delta \pi_{II}$	9.4949
$\Delta \pi_{II}$	0″.31
$\log \Delta i$	0.4433
Δi	2″.78

Ω_1	303°	38′	29″.9
π_1	121	16	53 .9
i_1	10	49	41 .7

An Ephemeris of a Comet.

In order to compute the right ascension and declination of a comet for a given date, we must find the true anomaly. Since in this case we are to deal with a parabola, the formulæ of Art. 11 do not apply. Kepler's laws, the truth of one of which was assumed in Art. 10, will first be proven.

23. *The Law of Areas; Kepler's Second Law.* Kepler's second law states that "the radius vector of each planet describes equal areas in equal times." It will be shown that this law is true, if Newton's law of gravitation is true: the latter is considered as established by experience.

Let a planet or comet be at P and the Sun at O; the mass of the Sun is taken as unity and that of the planet or comet as m. m is therefore a very small number. The mutual attraction between the two bodies gives each an acceleration; the pull exerted on the Sun is equal in magnitude to that exerted on the planet. When the distance between them is a unit [the semi-major axis of the Earth's orbit (See Art. 1)], let the acceleration given to the planet be k^2. Since the Sun's mass is $\frac{1}{m}$ times the planet's mass, the acceleration given to the Sun is mk^2. Hence the total accel-

eration, which, because it tends to diminish the distance between them, is accounted negative, is $-k^2(1+m)$. Since the force of attraction varies inversely as the square of the distance between the two bodies, the acceleration varies in the same ratio; when this distance is r the acceleration becomes $\frac{-k^2(1+m)}{r^2}$. In Fig. (9) let O be the Sun's position and P the planet's. POM $= v$, x and y being the co-ordinates of P; the xy-plane is the plane in which the body continually moves. By multiplying the acceleration along the line OP by cos v and sin v respectively we may resolve it into two components parallel to the axes of x and y.

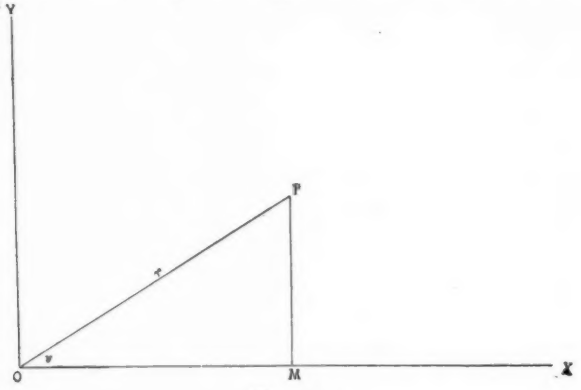


FIG. 9.

Since $\cos v = \frac{x}{r}$ and $\sin v = \frac{y}{r}$ we have

$$\frac{d^2x}{dt^2} = -\frac{k^2(1+m)}{r^2} \cdot \frac{x}{r} \qquad (58)$$

$$\frac{d^2y}{dt^2} = -\frac{k^2(1+m)}{r^2} \cdot \frac{y}{r} \qquad (59)$$

Multiplying (58) by y and (59) by x and subtracting the results we have

$$x\frac{d^2y}{dt^2} - y\frac{d^2x}{dt^2} = 0$$

the integration of which gives (t being the independent variable)

$$x\frac{dy}{dt} - y\frac{dx}{dt} = c. \qquad (60)$$

The value of the constant c will be determined hereafter. Since $x = r\cos v$ and $y = r\sin v$, differentiation gives

$$\left.\begin{aligned} dx &= \cos v\, dr - r\sin v\, dv \\ dy &= \sin v\, dr + r\cos v\, dv \end{aligned}\right\} \qquad (61)$$

By substituting these values of x, y, dx *and* dy in (60) we obtain after reduction

$$r^2dv = cdt. \tag{62}$$

Equation (62) shows that the radius vector of a planet sweeps over equal areas in equal times; for if in a time dt the radius vector sweeps through an angle dv, the area of the sector which it generates is $\frac{1}{2}r^2dv$. This area therefore equals $\frac{c}{2}dt$. Since $\frac{c}{2}$ is a constant the sectorial area has a constant ratio to the time in which it is described.

24. *Form of the Orbit; Kepler's First Law.*—Multiplying (58) and (59) respectively by $2\frac{dx}{dt}$ and $2\frac{dy}{dt}$ and adding the products, we obtain by transposition

$$2\left\{\frac{dx}{dt}\cdot\frac{d^2x}{dt^2}+\frac{dy}{dt}\cdot\frac{d^2y}{dt^2}\right\}+\frac{2k^2(1+m)}{r^3}\left\{x\frac{dx}{dt}+y\frac{dy}{dt}\right\}=0. \tag{63}$$

Differentiation of the equation $r^2 = x^2 + y^2$ gives

$$r\frac{dr}{dt} = x\frac{dx}{dt} + y\frac{dy}{dt}. \tag{64}$$

Combining (63) and (64) we have

$$\frac{d\left\{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2\right\}}{dt}+\frac{2k^2(1+m)}{r^2}\frac{dr}{dt}=0. \tag{65}$$

Multiplying through by dt and integrating we obtain (the unknown constant being called h)

$$\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2-\frac{2k^2(1+m)}{r}+h=0. \tag{66}$$

Squaring (60)

$$c^2 = x^2\left(\frac{dy}{dt}\right)^2 + y^2\left(\frac{dx}{dt}\right)^2 - 2xy\frac{dx}{dt}\cdot\frac{dy}{dt}$$

$$= (x^2+y^2)\left\{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2\right\}-\left\{x\frac{dx}{dt}+y\frac{dy}{dt}\right\}^2. \tag{67}$$

Combining (67) and (64) and substituting r^2 for $x^2 + y^2$ we obtain

$$\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2-\left(\frac{dr}{dt}\right)^2=\frac{c^2}{r^2}. \tag{68}$$

The subtraction of (68) from (66) gives

$$\left(\frac{dr}{dt}\right)^2=\frac{1}{r^2}\left\{-hr^2+2k^2(1+m)\,r-c^2\right\};$$

whence

$$dt = \frac{rdr}{\sqrt{-hr^2 + 2k^2(1+m)\,r - c^2}}. \qquad (69)$$

From (69) and (62) results

$$dv = \frac{cdr}{\sqrt{-hr^2 + 2k^2(1+m)\,r - c^2}}. \qquad (70)$$

In order to transform (70) so that it will conform to the ordinary nomenclature used in analytic geometry we proceed as follows. Let

$$h = \frac{k^2(1+m)}{a}, \qquad (71)$$

and

$$c^2 = a(1-e^2)\,k^2(1+m). \qquad (72)$$

Equation (70) then becomes

$$dv = \frac{a\sqrt{1-e^2}\,dr}{r\sqrt{2ar - r^2 - a^2(1-e^2)}}.$$

Multiplying numerator and denominator by $\frac{\sqrt{1-e^2}}{er}$ we obtain

$$dv = \frac{\frac{-a(1-e^2)\,dr}{er^2}}{\sqrt{1-\left\{\frac{a(1-e)^2}{er} - \frac{1}{e}\right\}^2}}. \qquad (73)$$

Since the numerator is the differential of the quantity in parenthesis in the denominator, it is evident that the entire fraction is in the form of the differential of an inverse cosine. The integration of (73) gives (introducing λ as the constant of integration)

$$v = \cos^{-1}\left(\frac{1}{e}\left\{\frac{a(1-e^2)}{r} - 1\right\}\right) + \lambda;$$

whence

$$\cos(v-\lambda) = \frac{1}{e}\left(\frac{a(1-e^2)}{r} - 1\right). \qquad (74)$$

From (74) is derived

$$r = \frac{a(1-e^2)}{1 + e\cos(v-\lambda)}. \qquad (75)$$

We have now obtained the familiar polar equation of a conic section, in which the origin is at one focus; a denotes the semi-major axis, e the eccentricity, and $v - \lambda$ the angle made by r, the radius vector, with that portion of the major axis between that focus which is chosen as origin and the nearest point on the curve. To determine the value of λ we take the special case in which a planet, revolving in an elliptical orbit, is at perihelion;

then $r = a\ (1 - e)$. By substituting this value of r in (75), $\cos\ (v - \lambda)$ becomes equal to 1, whence $v - \lambda = 0$. If OX in Fig. (9) be assumed to run through the perihelion, $v = 0$ when the planet is at perihelion, and therefore $\lambda = 0$; v has the same signification as in Art. 9. It is to be observed that Kepler's first law—the orbit of each planet is an ellipse, having the Sun at one of its foci—is a special case of an elliptical orbit embraced in the general formula (75).

25. *The Numerical Value of k.*—From (62) and (72) we derive

$$r^2dv = \sqrt{a(1-e^2)}\ k\ \sqrt{1+m}\ dt. \qquad (76)$$

By Art. 10 when $t = 0$, $v = 0$; when $t = \mathrm{T}$, $v = 2\pi$.

$$\therefore \int r^2dv = \int \sqrt{a(1-e^2)}\ k\ \sqrt{1+m}\ dt. \qquad (77)$$

To obtain the value of k we consider a planet's orbit. Since r^2dv is double the area of the sector generated by the radius vector in the time dt, $\int_{v=0}^{v=2\pi} r^2dv$ is double the area of the entire elliptical orbit described in one revolution; this double area $= 2\pi ab = 2\pi a^2\sqrt{1-e^2}$.

$$\int_0^{\mathrm{T}} \sqrt{a(1-e^2)}\ k\ \sqrt{1+m}\ dt = \sqrt{a(1-e^2)}\ k\ \sqrt{1+m}\ \mathrm{T}.$$

$$\therefore 2\pi a^2\sqrt{1-e^2} = \sqrt{a(1-e^2)}\ k\ \sqrt{1+m}\ \mathrm{T}.$$

Therefore

$$k = \frac{2a^{\frac{3}{2}}\pi}{\mathrm{T}\sqrt{1+m}} \qquad (78)$$

Knowing a, T and m for any particular planet in the solar system we may at once obtain the value of k. In the case of the Earth we take $a = 1$, $\mathrm{T} = 365.2563835$ mean solar days and $m = \frac{1}{354710}$, these data being given by Gauss in his *Theoria Motus*. From equation (78) we then have

$$\log k = 8.2355814 - 10.$$

26. *Kepler's Third Law.*—Let a, T and m be respectively the semi-major axis, periodic time, and mass of one planet in the solar system, which a_1, T_1 and m_1 are the corresponding quantities for another. From (78) we then have

$$\frac{2a^{\frac{3}{2}}\pi}{\mathrm{T}\sqrt{1+m}} = \frac{2a_1^{\frac{3}{2}}\pi}{\mathrm{T}_1\sqrt{1+m_1}};$$

whence

$$T^2 : T_1^2 :: \frac{a^3}{1+m} : \frac{a_1^3}{1+m_1}. \qquad (79)$$

Equation (79) shows that Kepler's third law—that the squares of the periodic times of two planets are to each other as the cubes of their mean distances from the Sun—is not quite accurate; for the denominators of the third and fourth terms of the proportion are not equal to unity: For Jupiter, the largest planet, the value of m is $\frac{1}{1047.355}$; for Mercury, the smallest of the major planets, it is $\frac{1}{6,000,000}$.

27. *The Elements of a Parabolic Orbit.* The elements of a parabolic orbit are

1. T, the time of passing perihelion.
2. ω, the distance of the perihelion from the node.
3. Ω, the longitude of the node.
4. i, the inclination of the orbit plane to that of the ecliptic.
5. $\log q$, the logarithm of the least distance from the Sun,—when the comet is at perihelion.

For explanations of ω, Ω and i see Art. 4. As an example we give the elements of the orbit of the comet 1900 *b*.

T 1900 August 3.20726 Greenwich Mean Time.

ω	12°	26′	13″.2	1900.0
Ω	328	0	30 .1	
i	62	30	46 .3	
$\log q$	0.006390			

28. *The True Anomaly and Radius Vector in a Parabolic Orbit.* In Analytic Geometry it is shown that p, the semi-latus rectum of a conic section, equals $a\,(1-e^2)$. Hence from (77) we obtain

$$\int r^2 dv = k\sqrt{p\,(1+m)}\;t. \qquad (80)$$

Most comets move in orbits which are sensibly parabolic; it is the custom of astronomers when computing the orbit of a newly discovered comet, to assume that it is an exact parabola; in most cases no certain deviation from this curve is shown by observation. We therefore proceed to derive formulæ needed in computing a parabolic orbit. Since $p = a\,(1-e^2)$, and e in the denominator of the second member of (75) becomes unity for a parabola, that equation may be written

$$r = \frac{p}{1 + \cos v} = \frac{p}{2\cos^2 \frac{1}{2} v} = q \sec^2 \frac{1}{2} v. \qquad (81)$$

$$\therefore r^2 dv = \frac{p^2}{4} \sec^4 \frac{1}{2} v dv = \frac{p^2}{2} \left(1 + \tan^2 \frac{1}{2} v\right) d\left(\tan \frac{1}{2} v\right) \qquad (82)$$

Combining (80) and (82) we have

$$k \sqrt{p\,(1 + m)}\, t = \frac{p^2}{2} \left(\tan \frac{1}{2} v + \frac{1}{3} \tan^3 \frac{1}{2} v\right). \qquad (83)$$

When the comet is at perihelion, both t and $v = 0$, and the integration constant in (83) must be 0; it is therefore omitted. The mass of a comet is so small in comparison with that of the Sun that it may be neglected; if q is the distance of the comet from the Sun at perihelion, $p = 2q$; equation (83) may then be written

$$\tan^3 \frac{1}{2} v + 3 \tan \frac{1}{2} v = \frac{3}{\sqrt{2}} \frac{kt}{q^{\frac{3}{2}}}. \qquad (84)$$

The elements of a parabolic orbit include log q and the time when the planet is at perihelion. To find v for any given date, we find the number of mean solar days between the time of passing perihelion and the given date and substitute this value for t in equation (84), the second member of which is thus completely known. If the time of passing perihelion were, for example, September 5.74,1900, the value of t for October 1.5 would be + 25.76. When the given date is before the time of passing perihelion t is negative. Thus for August 23.5 in the instance given above, $t = -13.24$.

Equation (84) may be solved as follows, unless special tables for its solution are at hand, such as are given in Oppolzer's *Lehrbuch zur Bahnbestimmung* or Watson's *Theoretical Astronomy*.

Let

$$\tan \frac{1}{2} v = 2 \cot 2\gamma = \cot \gamma - \tan \gamma.$$

Then

$$\tan^3 \frac{1}{2} v = \cot^3 \gamma - 3 \cot^2 \gamma \tan \gamma + 3 \cot \gamma \tan^2 \gamma - \tan^3 \gamma$$

$$= -3 \tan \frac{1}{2} v + \cot^3 \gamma - \tan^3 \gamma.$$

Combining this with (84) we have

$$\cot^3 \gamma - \tan^3 \gamma = \frac{3kt}{\sqrt{2}\, q^{\frac{3}{2}}}.$$

Placing $\cot \gamma = \sqrt[3]{\cot \frac{1}{2}\beta}$ we obtain

$$\frac{1 - \tan^2 \frac{1}{2}\beta}{\tan \frac{1}{2}\beta} = \frac{3kt}{\sqrt{2}q^{\frac{3}{2}}}.$$

whence

$$\tan \beta = \frac{2^{\frac{3}{2}}}{3k} \cdot \frac{q^{\frac{3}{2}}}{t}.$$

When q and t are known, β is first computed, its absolute value being always less than 90°; if t is negative, β is negative. γ is next found; it has the same sign as β, and its absolute value is likewise less than 90°. The value of $\frac{1}{2}v$ is found from $\tan \frac{1}{2}v = 2 \cot 2\gamma$. If the planet has passed perihelion, so that t is positive, $\frac{1}{2}v$ is also positive and less than 90°. Before perihelion, t being negative, $\frac{1}{2}v$ is also negative and lies between 0° and — 90°. In illustration we now proceed to compute v for November 18.5, 1900, using the elements given in Art. 27. Let

$$c = \frac{cq^{\frac{3}{2}}}{3k}.$$

$$\therefore \tan \beta = \frac{cq^{\frac{3}{2}}}{t}.$$

t	107.29274	$\log \cot \gamma$	0.203382
$\log q^{\frac{3}{2}}$	0.009585	γ	32° 2′ 57″.0
$\log c$	1.738842	2γ	64 5 54 .0
$\operatorname{colog} t$	7.969430	$\log \cot 2\gamma$	9.686287
$\log \tan \beta$	9.717857	$\log 2$	0.301030
β	27° 34′ 28″.8	$\log \tan \frac{1}{2}v$	9.987317
$\frac{1}{2}\beta$	13 47 14 .4	$\frac{1}{2}v$	44° 9′ 48″.6
$\log \cot \frac{1}{2}\beta$	0.610145	v	88 19 37 .2

29. *The Computation of an Ephemeris from Parabolic Elements.* The equator constants log sin a, log sin b, log sin c, A,

B and C are computed as in Art. 16. By the process of Art. 27, v is found. From Art. 17 $A' = A + \omega$, $B' = B + \omega$, and $C' = C + \omega$. Combining (44), (45) and (46) with (81) we have

$$x' = q \sin a \sin (A' + v) \sec^2 \frac{1}{2} v$$

$$y' = q \sin b \sin (B' + v) \sec^2 \frac{1}{2} v$$

$$z' = q \sin c \sin (C' + v) \sec^2 \frac{1}{2} v.$$

The values of ξ, η and ζ are next found as in Art. 19; α, δ and $\log \rho$ are obtained as in Art. 20. Finally the process of Art. 21 enables us to find the *true* values of α and δ. As all these computations have already been illustrated in full in the case of Eros, it will not be necessary to give similar computations for the comet.

30. *The Aberration Time.* Often in an ephemeris there is given for each date the value of the "aberration time," which is the time required for light to travel from the planet or comet to the Earth. Since light consumes 498.6 seconds in traversing a distance of unity (the mean distance of the Earth from the Sun), the time required for any other known distance is found by multiplying 498.6 by that distance. For example, if at a given time $\log \rho = 0.30103$, the aberration time would be 997^s at that instant.

31. *The Brightness.* In the case of a comet the brightness at the time of discovery is taken as the unit. Let r and ρ be the distances of the comet from the Sun and Earth respectively at the time of discovery, and r_1 and ρ_1 be the corresponding distances at some other time. Let I be the intensity of sunlight at a distance of unity from the Sun; then as the intensity of light varies inversely as the square of the distance—assuming that the light of a comet is due wholly to reflected sunlight—we find that its intensity when it strikes the comet is $\frac{I}{r^2}$. As the comet is not a mirror, the reflected light is scattered in all directions, and consequently follows the law of radiated light—that is, its intensity varies inversely as the square of the distance. Since only a portion of the light is reflected, we may consider the intensity changed to another value $\frac{I'}{r^2}$ at a units distance from the comet. Therefore when it has reached the Earth at a distance of ρ, its intensity

will be $\frac{I'}{r^2\rho^2}$. For any other distances as r_1 and ρ_1 we obtain an intensity equal to $\frac{I'}{r_1^2\rho_1^2}$. The intensity at the second time is to the intensity at the time of discovery as $r^2\rho^2$ is to $r_1^2\rho_1^2$; hence if the brightness at the time of discovery is denoted by unity, as stated above, the second brightness will be denoted by $\frac{r^2\rho^2}{r_1^2\rho_1^2}$. Owing to the capricious changes in the brightness of a comet, arising from causes imperfectly understood, this formula does not give absolutely accurate values.

The brightness of a minor planet is measured on the ordinary scale of stellar magnitudes. Like Mars, each asteroid must be gibbous most of the time; but as an asteroid is usually observed when near its opposition, its gibbousness is then so slight that we shall not consider it.

Let J_0 be the brightness of an asteroid when its distance from the Sun is r_0 and from the Earth is ρ_0; also let J be the brightness at the corresponding distances r and ρ. Then, by reasoning similar to that applied to the comet, we obtain

$$J = J_0 \frac{r_0^2\rho_0^2}{r^2\rho^2}.$$

As the unit of brilliancy we take the brightness of the asteroid at a distance of a (the semi-major axis of its orbit) from the Sun and $a - 1$ from the Earth. Substituting these values for r_0 and ρ_0, and unity for J_0 we have

$$J = \frac{a^2(a-1)^2}{r^2\rho^2}$$

$$\text{or } \log J = 2\log[a(a-1)] - 2\log(r\rho).$$

Let m_0 be the magnitude of the planet when $r = a$ and $\rho = a - 1$ and M be its magnitude at any other time. The scale of magnitudes is so chosen that an object of the fifth magnitude is $\sqrt[5]{100}$ times as bright as one of the sixth, while one of the fourth magnitude is $\sqrt[5]{100}$ times as bright as one of the fifth, etc. [See Young's *General Astronomy* Art. 819]. Let the magnitude be m_0 when J is unity; then when the magnitude is M

$$J = (\sqrt[5]{100})^{m_0 - M};$$

whence

$$\log J = 0.4(m_0 - m) = (m_0 - M)\log \sqrt[5]{100}$$

or

$$M = m_0 - 2.5 \log J.$$

Substituting for log J the value previously obtained we have

$$M = m_0 + 5 \log (r\rho) - 5 \log (a^2 - a).$$

Denoting by g the constant quantity $m_0 - 5 \log (a^2 - a)$, we have

$$M = g + 5 \log (r\rho).$$

An observer who estimates the magnitude of the planet at a time when r and ρ are known may thus find the value of the constant g. Having determined g, the same equation may be used for computing the magnitude at any other time when r and ρ are known. For example, an observer estimated that Eros, on a particular night when $\log (r\rho) = 0.08$, was of magnitude 11.0. Substituting this value in the last formula we have $g = 10.6$. Then by using the values of $\log r$ and $\log \rho$ given in Arts. 12 and 20 we have $M = 8.5$ for December 25.5, 1900.

SPECTROSCOPIC NOTES.

The death of Professor Henry A. Rowland, of Johns Hopkins University, occurred in Baltimore April 16. Professor Rowland's work has been rather in physics than in astronomy; but his values of wave-lengths of lines in the solar spectrum have been so fundamental and so accurate that at the present time spectroscopic results are usually expressed in terms of them; and his gratings form an indispensable part of every spectroscopic equipment, and have done splendid service in some of the most notable of modern astronomical investigations. Although the product of his work has been universally used in spectroscopic measurement for a considerable period of time, and although he has been the instructor of a large number of the younger physicists and astronomers of the present time, yet Professor Rowland has now passed away at the early age of fifty-two.

In the *Astrophysical Journal* for March Professor Lord presents the results of a discussion of the two photographs of the flash spectrum which he obtained at the last eclipse as a member of the Washington Observatory eclipse party. His instrument was a large star spectroscope, with two dense equilateral prisms, attached to a 4-inch telescope. Considerable deviations are shown in the list of lines in the flash spectrum from the catalogue of chromosphere lines. While all the bright lines of the flash spectrum except the helium lines are identified with dark lines in the solar spectrum, many of the dark lines in the solar spectrum seem not to appear in the flash spectrum, and the relative intensities of the bright lines of the flash spectrum do not agree with the relative intensities of the dark lines in the solar spectrum.

Mr. Evershed (*Proceedings of the Royal Society*, No. 442), as the result of the study of his photographs secured at the total eclipse of Jan. 22, 1898 in India, has found that while the relative intensities of the bright lines in the flash spectrum and of the dark lines in the solar spectrum are not the same, the lines due to any one element preserve the same relative intensities in the two cases; the variation of the relative intensities of the lines for different elements being

presumably due to differences in the altitudes above the Sun's photosphere at which the vapors of these elements occur. Twenty-eight hydrogen lines were measured, the series agreeing well with Balmer's formula except in the case of *h*, which was displaced to the red. A new corona line was discovered at λ 3388 in the ultra-violet. The position deduced for the green corona line is in agreement with recent results.

Señor Ascarza (*Astronomische Nachrichten*, 3698) at the eclipse of May 1900, observed visually the spectrum of the corona with a three-prism spectroscope attached to a 5-inch telescope. In addition to the continuous spectrum the green corona line was seen, and a micrometer setting was made, the reduction of which gives for the position of the line in the spectrum λ 5298.7.

The star BD + 43°726, whose variability is established by the observations of Mr. Williams (*Astronomische Nachrichten*, 3698), is entered as red, with a spectrum of 'type IV!' in Mr. Espin's list of stars with remarkable spectra.

The spectrum of Nova Persei for the first two days, while it was brightest, was of the first or hydrogen type. A change in color from white to yellow or red accompanied the complete change in character of spectrum to the usual type for new stars, with bright hydrogen and other lines, in this case extraordinarily broad, having dark components toward the violet.

PLANET NOTES FOR JUNE AND JULY.

H. C. WILSON.

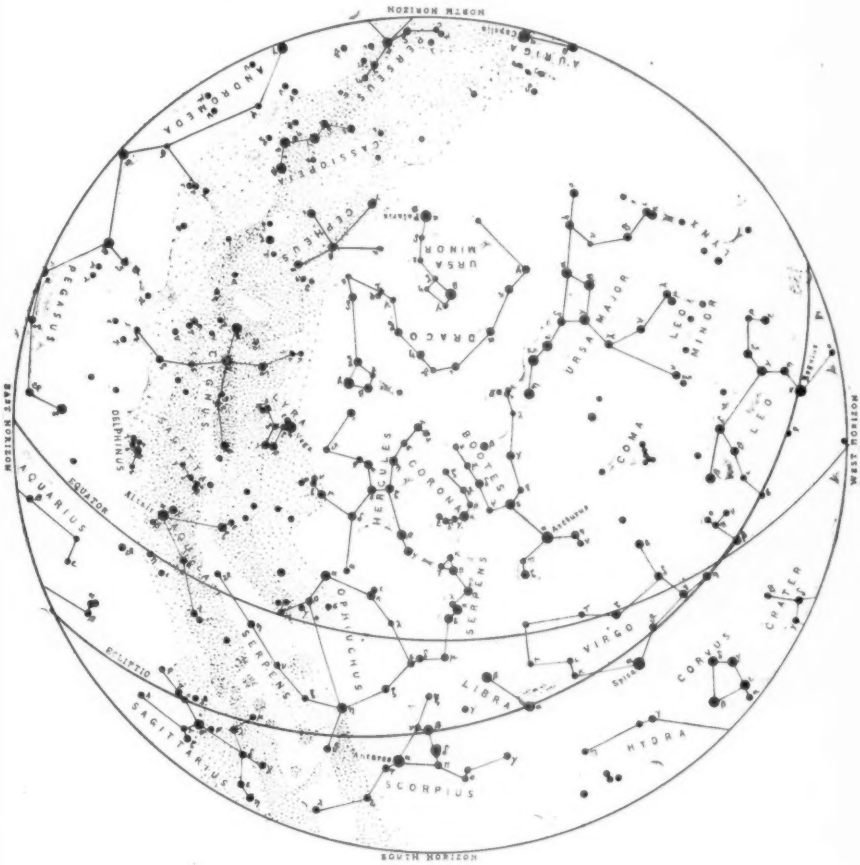
Mercury will be at greatest elongation, east from the Sun 24° 39′, on June 15, and will therefore be visible as evening star during the middle of June. It returns to inferior conjunction July 13 and to greatest western elongation Aug. 2.

Venus is evening star, but quite close to the Sun as yet. During the latter part of the month and in July it will not be difficult for anyone to pick up the planet with the naked eye about an hour after sunset. Look close to the horizon and near the sunset point. On the night of June 30 the two planets Venus and Mercury will be in conjunction and it may be possible to see both at the same time, with the naked eye or with a telescope of low power. Mercury will then be about four degrees south of Venus. The Moon will be in conjunction with Venus on June 17 and July 17 and with Mercury June 18 and July 15.

Mars, as anyone can now readily see, is moving eastward and will soon leave behind the constellation Leo in which it has tarried all of this year. It will continue to be visible in the evening during the summer, but its position is gradually becoming less favorable for observation, both because of its increasing distance from the Earth and the lessening altitude at which it can be observed. The Moon will be in conjunction with Mars on June 22 and July 21.

Jupiter and *Saturn* are to be the summer planets, Jupiter coming to opposition June 30 and Saturn July 5. The two look like a pair of twins now, being close together in Sagittarius, and both being far more conspicuous than any of the stars in that region of the sky. Saturn's golden color and Jupiter's white

and more brilliant light furnish a contrast pleasing to the eye. Unfortunately on account of the low altitudes of the planets, the telescopic views of their markings will on most occasions be disappointing, yet generally one will without difficulty distinguish the more prominent features. The wonderful rings of Saturn are now turned at an angle of about 25° to the line of sight, so that they can be separated clearly when the seeing is good. The Moon will be in conjunction with the two planets June 2, July 1 and July 28.



THE CONSTELLATIONS AT 9 P. M., JULY 1, 1901.

Uranus may be seen with a telescope about midnight in Scorpio, ten degrees to the northeast from Antares. This planet comes to opposition June 6 and will be in conjunction with the Moon on June 2, June 29 and July 26.

Neptune will be in conjunction with the Sun on June 20 and so will not be visible during the summer.

The Moon.

		Phases.	Rises. (Central Standard Time at Northfield Local Time 13m less). h m	Sets. h m
June	2	Full Moon	8 13 P. M.	5 34 A. M.
	9	Last Quarter	11 55 "	11 59 "
	16	New Moon	4 39 A. M.	8 9 P. M.
	23	First Quarter	12 18 P. M.	11 59 "
July	1	Full Moon	7 42 "	5 21 A. M.
	8	Last Quarter	11 19 "	1 24 P. M.
	15	New Moon	4 32 A. M.	7 32 "
	23	First Quarter	1 06 P. M.	11 24 "
	31	Full Moon	7 31 "	6 28 A. M.

Occultations Visible at Washington.

Date. 1901.		Star's Name.	Magnitude.	IMMERSION. Washington M.T. h m	IMMERSION. Angle from N pt. °	EMERSION. Washington M. T. h m	EMERSION. Angle from N pt. °	Duration. h m
June	1	ω Ophiuchi	4.7	10 04	174	10 29	207	0 25
	3	15 Sagittarii	5.5	7 58	94	9 03	271	1 04
	3	17 Sagittarii	7.0	8 49	45	9 37	319	0 48
	3	21 Sagittarii	4.9	14 12	87	15 37	248	1 25
	4	*d* Sagittarii	4.9	13 39	134	14 24	195	0 45
	4	ρ^2 Sagittarii	6.1	16 33	54	17 46	266	1 13
	9	15 Piscium	6.6	12 33	70	13 32	246	0 59
	27	B.A.C. 5109	5.4	14 49	113	15 44	251	0 55
	29	B.A.C. 5758	6.6	8 08	107	9 34	266	1 26
July	1	Lalande 35497	6.4	14 20	76	15 37	249	1 17
	12	107 Tauri	6.5	14 22	127	14 58	219	0 36
	25	λ Libræ	5.0	6 40	90	8 12	394	1 32
	29	B.A.C. 6658	7.3	7 39	88	8 58	254	1 19

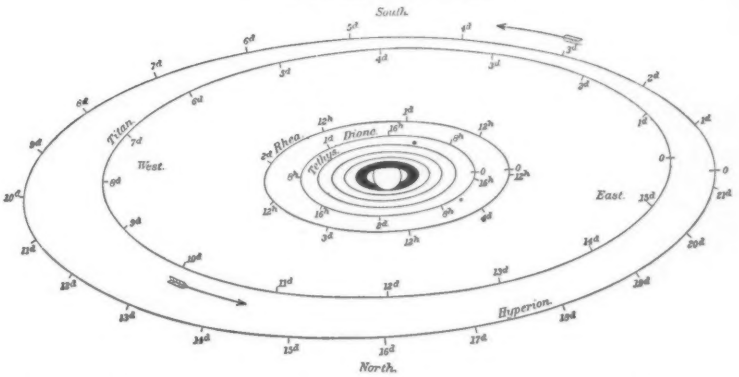
Phenomena of Jupiter's Satellites.

Central Standard Time.

		h	m				h	m		
July	2	11	13 P. M.	II	Ec. Re.	July 14	1	00 A. M.	I	Ec. Re.
	3	8	02 "	III	Tr. Eg.		7	32 P. M.	I	Tr. In.
		8	27 "	III	Sh. Eg.		7	53 "	I	Sh. In.
	5	11	22 "	I	Tr. In.		9	49 "	I	Tr. Eg.
		11	30 "	I	Sh. In.		10	10 "	I	Sh. Eg.
	6	1	39 A. M.	I	Tr. Eg.	15	7	29 "	I	Ec. Re.
		1	47 "	I	Sh. Eg.	17	12	50 A. M.	II	Oc. Dis.
		8	41 P. M.	I	Oc. Dis.		4	25 "	II	Ec. Re.
		11	05 "	I	Ec. Re.		11	39 P. M.	III	Tr. In.
		11	12 "	IV	Oc. Dis.	18	1	21 A. M.	III	Sh. In.
	7	2	51 A. M.	IV	Ec. Re.		2	42 "	III	Tr. Eg.
		8	05 P. M.	I	Tr. Eg.		4	28 "	III	Sh. Eg.
		8	16 "	I	Sh. Eg.		7	53 P. M.	II	Tr. In.
	9	10	34 "	II	Oc. Dis.		8	46 "	II	Sh. In.
	10	1	49 A. M.	II	Ec. Re.		10	41 "	II	Tr. Eg.
		8	20 P. M.	III	Tr. In.		11	35 "	II	Sh. Eg.
		9	22 "	III	Sh. In.	20	2	50 A. M.	I	Tr. In.
		11	24 "	III	Tr. Eg.		3	19 "	I	Sh. In.
	11	12	27 A. M.	III	Sh. Eg.	21	12	10 "	I	Oc. Dis.
		8	45 P. M.	II	Tr. Eg.		2	55 "	I	Ec. Re.
		8	59 "	II	Sh. Eg.		9	17 P. M.	I	Tr. In.
	13	1	06 A. M.	I	Tr. In.		9	48 "	I	Sh. In.
		1	24 "	I	Sh. In.		11	34 "	I	Tr. Eg.
		3	23 "	I	Tr. Eg.	22	12	05 A. M.	I	Sh. Eg.
		3	42 "	I	Sh. In.		9	23 P. M.	I	Ec. Re.
		10	25 P. M.	I	Oc. Dis.					

NOTE.—In. denotes ingress; Eg., egress; Dis., disappearance; Re., reappearance; Ec., eclipse; Oc., occultation; Tr., transit of the satellite; Sh., transit of the shadow.

The Satellites of Saturn.



APPARENT ORBITS OF THE SEVEN INNER SATELLITES OF SATURN, AT OPPOSITION IN 1901 AS SEEN IN AN INVERTING TELESCOPE.

I. MIMAS.

Period $0^d\ 22.6^h$

		h		
July	1	7.3	P. M.	E
	4	3.8	A. M.	W
	5	2.4	"	W
	6	1.0	"	W
	6	11.7	P. M.	W
	7	10.3	"	W
	8	8.9	"	W
	9	7.5	"	W
	12	4.0	A. M.	E
	13	2.6	"	E
	14	1.3	"	E
	14	11.9	P. M.	E
	15	10.5	"	E
	16	9.1	"	E
	17	7.7	"	E
	21	2.9	A. M.	W
	22	1.5	"	W
	23	12.1	"	W
	23	10.7	P. M.	W
	24	9.3	"	W
	25	7.9	"	W
	29	3.1	A. M.	E
	30	1.7	"	E
	31	12.3	"	E
	31	10.9	P. M.	E

II. ENCELADUS.

Period $1^d\ 8.9^h$

		h		
July	1	5.5	P. M.	E
	3	2.4	A. M.	E
	4	11.3	"	E
	5	8.1	P. M.	E
	7	5.0	A. M.	E
	8	1.9	P. M.	E
	9	10.8	"	E
	11	7.7	A. M.	E

ENCELADUS, Cont.

		h		
July	12	4.5	P. M.	E
	14	1.4	A. M.	E
	15	10.3	"	E
	16	7.2	P. M.	E
	18	4.0	A. M.	E
	19	12.9	P. M.	E
	20	9.8	"	E
	22	6.7	A. M.	E
	23	3.5	P. M.	E
	25	12.4	A. M.	E
	26	9.3	"	E
	27	6.2	P. M.	E
	29	3.1	A. M.	E
	30	11.9	"	E
	31	8.8	P. M.	E

III. TETHYS.

Period $1^d\ 21.3^h$

		h		
July	1	8.1	P. M.	E
	3	5.4	"	E
	5	2.7	"	E
	7	11.9	A. M.	E
	9	9.2	"	E
	11	6.5	"	E
	13	3.8	"	E
	15	1.1	"	E
	16	10.4	P. M.	E
	18	7.7	"	E
	20	5.0	"	E
	22	2.3	"	E
	24	11.5	A. M.	E
	26	8.8	"	E
	28	6.1	"	E
	30	6.4	"	E
	31	12.7	"	E

IV. DIONE.

Period $2^d\ 17.7^h$

		h		
July	2	1.8	A. M.	E
	6	7.5	P. M.	E
	8	1.1	"	E
	11	6.8	A. M.	F
	14	12.4	"	E
	16	6.1	P. M.	E
	19	11.7	A. M.	E
	22	5.4	"	E
	24	11.0	P. M.	E
	27	4.7	"	E
	30	10.3	A. M.	E

V. RHEA.

Period $4^d\ 12.5^h$

		h		
June	29	8.1	P. M.	E
July	4	8.4	A. M.	E
	8	8.7	P. M.	E
	13	9.0	A. M.	E
	17	9.4	P. M.	E
	23	9.7	A. M.	E
	26	10.0	P. M.	E
	31	10.0	A. M.	E

VI. TITAN.

Period $15^d\ 23.3^h$

		h		
June	30	11.8	P. M.	E
July	5	1.1	A. M.	I
	9	1.9	"	W
	12	11.2	P. M.	S
	16	9.3	"	E
	20	10.7	"	I
	24	11.7	"	W
	28	7.8	"	S
Aug.	1	6.7	"	E

VII. HYPERION. Period 29d 7.6h		HYPERION, Cont. Period 29d 7.6h		VIII. IAPETUS. Period 79d 22.1h	
h		h		h	
July 1.8	I	July 22.9	I	June 27.6	E
7.6	W	28.8	W	July 16.5	I
13.6	S	Aug. 3.7	S	Aug. 5.8	W
18.3	E				

Ephemeris of Comet *a* 1901.—The following ephemeris given in *Astronomical Journal*, No. 498, was computed from Kreutz's elements. It shows that comet's position is becoming continually less favorable for observation:

1901	R. A. h m s	Decl. ° ′	log Δ	Brightness
May 28.5	7 33 49	+7 13	0.206	9.024
29.5	36 50	24		
30.5	39 46	35		
31.5	42 32	45		
June 1.5	6 45 14	55	0.239	0.018
2.5	47 51	8 5		
3.5	50 24	14		
4.5	52 51	23		
5.5	55 14	31	0.269	0.017
6.5	57 30	39		
7.5	59 43	47		
8.5	7 1 55	8 54		
9.5	4 04	+9 01	0.291	0.017

Brightness April 24 = 1.000.

Search Ephemeris for Comet 1894 I (Denning.)—In *Astronomische Nachrichten* No. 3700 Dr. P. V. Neugebauer gives a rough search ephemeris for the faint periodic comet discovered by Denning in 1894. The position of the comet is quite unfavorable for observation, but as the time of perihelion is somewhat uncertain there is a possibility of its being found at some distance from the predicted place. The portion of the ephemeris for the coming month is as follows:

Perihelion Passage		May 9 °	May 25 °	June 10 °
1901 May 25	α	111.2	100.6	91.0
	δ	+25.6	+25.6	+25.0
June 10	α	129.0	118.4	108.3
	δ	+22.7	+24.1	+24.7
26	α	145.3	135.6	125.7
	β	+18.2	+20.6	+22.4

Definitive Orbit of Comet 1894 II.—In the *Astronomical Journal*, No. 496-497, Mr. Henry A. Peck, of Syracuse University, Syracuse, N. Y., gives the results of a definitive determination of the orbit elements of Comet 1894 II, which was discovered by Mr. Walter F. Gale at Sydney, Australia, April 1, 1894. The published observations extending from April 2 to Aug. 21, 1894 were combined into 11 normal places, giving 22 normal equations from which the elements were determined.

$$T = 1894, \text{April } 13.406912 + 0.000395\, dv$$
$$\omega = 324^\circ\ 12'\ 22''.52 \quad + 1.2046\, dv \Big\}$$
$$\Omega = 306\ \ 23\ \ 53\ \ .04 \quad - 0.5347\, dv \Big\} 1894.0$$
$$i = \ \ 86\ \ 59\ \ 18\ \ .19 \quad + 0.8478\, dv \Big\}$$
$$q = 0.9830931 + 0.000001339\, dv$$
$$e = 0.9911206 + 0.000002837\, dv$$

The most probable orbit is thus a very long ellipse. The value of dv which gives the minimum sum of the squares of the residuals is — 40″, but this may vary 20″ each way without materially affecting the residuals. Assuming dv to be — 40″ the period of revolution is 1143 years.

"The orbit sustains four peculiar relations to that of Jupiter. During the entire time of visibility, and for two or three years previous, the planet was near the orbit plane. From a cosmological standpoint the comet's previous history is of considerable interest. The computation of the perturbations by the major planets previous to discovery is now in progress, and the results will be presented later.

Elements of Asteroid (314) Rosalia.—Herr Berberich gives the following elements in *Astronomische Nachrichten* No. 3708:

Epoch 1891 Sept. 2.5 Berlin M. T.		1899 Feb. 18.0
$M =$	1° 14′ 55″.6	123° 43′ 06″.5
$\omega =$	186 03 04 .7	185 36 38 .9
$\Omega =$	171 12 41 .5 1890.0	171 19 58 .2
$i =$	12 32 53 .9	12 32 30 .9
$\phi =$	10 47 50 .7	10 40 12 .8
$\mu =$	636″.406	636″.6909
$\log a =$	0.497514	0.4973853

Ephemeris of Asteroid (314).

	R. A.			Decl.			$\log r$	$\log \Delta$
	h	m	s	°	′			
May 27	16	29	05	— 4	05.4		0.5114	0.3551
29		27	33	3	59.6			
31		26	01	3	54.3		0.5105	0.3541
June 2		24	29	3	49.5			
4		22	58	3	45.1		0.5096	0.3539
6		21	27	3	41.3			
8		19	57	3	38.1		0.5086	0.3546
10		18	29	3	35.5			
12		17	03	3	33.4		0.5077	0.3560
14		15	39	3	31.9			
16		14	16	3	30.9		0.5067	0.3582
18		12	56	3	30.5			
20		11	40	3	30.7		0.5058	0.3611
22		10	27	3	31.5			
24	16	09	17	— 3	32.8		0.5048	0.3648

Magnitude 13.2. Probable error of position: ± 1^m; ± 1′.3.

GENERAL NOTES.

This number is for June and July, and the next issue will be for August and September and it will appear before the first of the last named month.

We have again added eight pages more than usual in order to print the entire article by Miss Traylor, which is a clear and concise statement of the method of computing an ephemeris of a planet or comet. College students in astronomy will certainly be interested in this paper.

A Floating Telescope.—In *Monthly Notices* for April, there appears a description by B. Cookson, of a new form of telescope, built at the Cambridge Observatory (England) which possesses some points of interest for the lovers of

celestial photography. This new form of telescope is floated in a trough of mercury. This trough is an annular vessel in which floats another body with about one-half inch of space in the clear. The telescope is mounted at the center of the float. The mercury necessary to do the work is about 125 lbs. which is some more than a gallon of the liquid. The report of the working of this new form of instrument is very favorable.

United States Naval Observatory.—The Board of Visitors appointed by the President and confirmed by the Senate was summoned not long ago by Secretary Long, of the Navy, to meet in Washington to look into the condition of the Naval Observatory. The Board is directed by law to investigate the condition of buildings, instruments, apparatus, prosecutions of scientific work, and to report upon the selection of astronomical director and director of the *Nautical Almanac*, which positions were left vacant by the recent detachment of Professor S. J. Brown. The law also gives the Board the power to investigate and report upon the efficiency with which the scientific work is prosecuted, and also in regard to all expenditures in the administration of the Observatory. Those powers are mandatory, and the Board has no discretion.

The criticism of the astronomers throughout the country that the output of scientific work is not commensurate with the liberal appropriations provided by Congress will be passed upon for the first time by an official Board and the responsibility for the alleged inefficiency of the Observatory will probably be located. The Board will also be required to draw up new regulations for the government of the Observatory.

The board is composed of Professor E. C. Pickering, of Harvard Observatory; Professor Chas. A. Young, professor of astronomy at Princeton; Dr. William R. Harper, president of the University of Chicago; Professor Ormond Stone, professor of astronomy at the University of Virginia; Dr. Charles F. Chandler, dean of the school of applied science, Columbia University, and Professor Asaph Hall, Jr., professor of astronomy at the University of Michigan.

In Memory of George Anderson.—The officials of the Naval Observatory have just erected an appropriate monument to mark the grave of George Anderson, in Arlington cemetery. He was connected with the Observatory for thirty-one years and was attached to the big telescope for twenty-seven years. He discovered some important double stars besides aiding in the discovery of the satellites of Mars, in 1877.

The monument is a granite column about five feet high and bears the inscription:

GEORGE ANDERSON,
Born June 21, 1839,
Died Oct. 31, 1900.
Corporal Troop I, 3d New York, Provisional Cavalry.
For 31 years a trusted servant of the U. S.
Naval Observatory.
He aided in the discovery of the satellites of Mars, Aug. 11-17, 1877.
A tribute from his astronomical associates.

George Anderson was born in Edinburgh, Scotland, and came to the United States in 1858. He resided in New York until the Civil War, when he enlisted in the service of the Government, and came to Washington. He saw hard service in Virginia and the Carolinas. At 'the close of the war he made the District of Columbia his home, and was attached first to the War Department, and after-

ward to the Naval Observatory, where his faithful work as an assistant on the great telescope won him the reward of Professors Hall, Newcomb, and Harkness. He aided the former in the discovery of the satellites of Mars.—*The Washington Post, Sunday, April 21, 1901.*

An Observational Determination of the Apparent Figure of the Sky.—It is a matter of common knowledge that the vault of the sky has an apparently flattened form, the horizon appearing to be more distant than the zenith. We know also, that a star group seen near the horizon looks much larger than when seen at a considerable altitude, and that, in a general way, the amount of enlargement diminishes with increasing altitude. The observations which follow were obtained with a view to determining quantitatively the amount of enlargement at various zenith distnnces, and thus to find the eccentricity of a vertical section of the sky, supposing it to be elliptical.

A series of estimates of the angular distances of stars was made by a class of Junior engineering students, all of whom were somewhat familiar with the estimation of angles. The reason for which the results were wanted was not mentioned until all the observations has been completed. A total of five hundred and fifty-two estimates was made, the distances assigned for observation being those from star to star in the "dipper" in Ursa Major, the "sickle" and the triangle $\delta\ \psi\ \beta$ in Leo, the four principal stars in Corvus, and five stars in Canis Major, chosen so as to afford a wide range in azimuth as well as in altitude. The standard of distance was the line α-β Geminorum, the middle point of which was at an altitude of 90°. The observations were made from the observing deck at the Observatory which commands a good horizon through 270° of azimuth.

The average probable error of a mean of estimates by the class is $\pm$ 0°.15. No estimates were rejected.

A summary of the results is given in table I, where the headings of the columns have the following significance:

d = computed distance.
d' = estimated "
h = altitude.
γ = the ratio of d' to d.
n = number of estimates of which d' is the mean.

Since angular dimensions vary inversely with the distance, the ratio of d' to d represents the apparent radius of the sky expressed in terms of the minor axis as unity.

TABLE I.

No.	d °	d' °	h °	γ	n
1	9.7	12.4	8	1.28	19
2	8.2	10.2	15	1.24	21
3	8.6	11.7	15	1.36	22
4	8.5	11.0	17	1.31	20
5	10.2	13.4	19	1.31	20
6	6.7	7.9	19	1.18	20
7	3.5	4.3	22	1.23	22
8	5.1	6.3	22	1.30	20
9	4.6	6.0	23	1.23	22
10	4.4	5.4	25	1.23	20
11	4.0	5.0	27	1.25	20
12	7.0	8.0	27	1.21	14
13	5.6	6.5	28	1.16	14

TABLE I.—Cont.

No.	d	d'	h	γ	n
	°	°	°		
14	5.3	6.4	29	1.21	14
15	5.4	6.4	30	1.18	20
16	10.2	12.0	31	1.18	20
17	3.5	3.9	32	1.11	14
18	4.4	5.1	33	1.16	20
19	7.9	9.1	38	1.18	20
20	5.2	6.1	42	1.17	20
21	4.0	4.8	42	1.20	20
22	5.1	6.0	44	1.18	20
23	3.5	4.1	46	1.14	20
24	6.6	7.6	50	1.17	20
25	2.8	2.7	51	0.97	20
26	4.9	5.3	65	1.08	14
27	4.7	4.7	68	1.00	14
28	3.5	3.9	72	1.10	14
29	6.6	7.2	75	1.09	14
30	3.0	2.8	76	0.93	14

The eccentricity may be expressed in terms of γ and h under the form

$$e = \frac{\sqrt{\gamma^2 - 1}}{\gamma \cos h}.$$

Grouping the quantities in table I with reference to h, and computing the value of e for each group, we have table II.

TABLE II.

h	γ	e	n
13′	1.293	0.649	62
21	1.256	0.651	144
30	1.183	0.612	136
42	1.182	0.713	80
49	1.093	0.606	60
71	1.040	0.837	70

The weighted mean of the values of e is, $e = 0.669 \pm 0.022$.

The corresponding ratio of minor to major axis is 0.743, or very nearly 3.4.

UNIVERSITY OF ILLINOIS OBSERVATORY, W. C. BRENKE.
April 20, 1901.

Comet Halls.—The comet of Australia, discovered by Halls on April 23d, 1901, was on display at the Mount Lowe Observatory, California, during 35 minutes, from $7^h\ 38^m$ to $8^h\ 13^m$, May 17th, 1901. Its approximate position at $7^h\ 38^m$ was

$$\alpha = 5^h\ 50^m\ 30^s$$
$$\delta = +5^\circ\ 46'$$

Motion rapid. The nucleus was bright and had the appearance of an orange colored star of the 4th magnitude, by estimation. The tail was invisible in the high power on micrometer and could not be measured. By time of transit over wire in a low power Gundlach periscopic eye-piece, its length was estimated at 5′ and its width was large in comparison.

The finding ephemeris was that of Professor Kreutz of Kiel Observatory. A spring fog hung round about the sumits of Tujunga for six days, and therefore the visitor from the south could not have been seen before.

The mists vanished the afternoon of May 17th and all was made ready for search at sunset. The ephemeris gave places for the 16th and 20th and for the 16th

$$\alpha = 5^h\ 44^m\ 4^s$$
$$\delta = +4^\circ\ 8'$$

Interpolating, the telescope was set, but brilliant twilight interferred at 7^h. With deepening shade, the nucleus was seen at $7^h\ 38^m$ and the tai was detected 10 minutes later, the display coming to an end at $8^h\ 13^m$ behind a rocky peak. The western horizon from the equator north is made up of the Verdugo and Tujunga ranges—low mountains—in altitude from 3° to 9°, which from the west round through the south, the mighty ocean presents a most admirable horizon. The purity of air here, its freedom from dust, aqueous vapor and varying currents and densities are such that one coming from the eastern observatories can scarcely fail being impressed with the decided improvement. The trapezium of Orion at times is seen, as it were, standing on rock on the smooth summits of the Sierra Madra mountains at the instant of rising. Half of the nebula may be above the surface, while the four stars are seen in apparent contact with the wind polished rock, without distortion. There was no time to secure a spectrum of the comet, and doubtless it would not have been of value if obtained so close to the summits of the mountains that rise near the sea. Very likely the tail of the comet would have appeared to be at least 1° in length if observed in a high altitude.

It is highly interesting to watch the changes in the Fraunhofer lines in the Brashear spectroscope, during an entire day, from the moment of the rising Sun over the arid summits of the Sierra Madra chain through all varying altitudes to meridian and to the time of disappearance in the distant saw-teeth of Tujunga. The changes in the number of the absorption lines are remarkable, especially when in January the Sun ignores the rock-crested mountains and retires to its abode in the ocean.

EDGAR L. LARKIN.

MOUNT LOWE OBSERVATORY,
May 18, 1901.

Modern Science and Christianity.—Dr. Talmadge of Washington gave in that city a most eloquent discourse, May 12, 1901, on the defeat of the materialists in the present march of Science, at the opening of the twentieth century. His showing was that the weapons of science with which the enemies of religion have been trying to destroy it, are being wrested from their hands and used against them for certain and signal defeat. The central thought was the sword of Goliath in the hands of David.

After giving illustrations of this fact from science in various lines of research, Dr. Talmadge came to the discoveries of astronomy, and touched upon the marvels of light discoveries and the story of Joshua. He remarked that "Worldly philosophy said, 'What an absurd story about Joshua making the Sun and Moon stand still. If the world had stopped an instant the whole universe would have been out of gear.' 'Stop,' said Christian philosophy, 'not quite so quick.' The world has two motions—one on its own axis, and the other around the Sun. It was not necessary in making them stand still, that both motions should be stopped—only the one turning the world on its own axis. There was no reason why the halting of the Earth should have jarred and disarranged the whole universe. Joshua right and God right; infidelity wrong every time; I knew it would be wrong. I thank God that the time has come when Christians need not be scared at any scientific exploration."

Dr. Talmadge's explanation may be sufficient but no one knows whether it is the true one or not. The command was that the Sun and Moon should stand still; he supposes the Earth to cease rotating, for the time being, which would produce the same result. It may be, however, that the Great Ruler of the Universe secured the result in some other way. The Bible student does not claim to know how, yet he believes the fact because it is supported by accredited testimony which he must receive.

Observations of Nova Persei, 1901.

Feb. 24, 7 to 9 P. M.—The "new star" is not fully as bright as Capella, but recalls Fomalhaut in size and color. Its trail obtained from 8:15 to 8:35 P. M. on the photographic plate of a small portrait lens shows it to be distinctly larger than Alpha Persei or than Algol at maximum. Nearly half Moon is present.

25, 7:30 P. M.—The brightness of the variable seems slightly decreased. The trails obtained from 8:20 to 8:50 shows less difference between it and Alpha Perseus than on the previous night. Half Moon.

26.—Brightness decreased as seen between moving clouds.

27. 8 P. M.—Equal to Alpha.

28. 9 P. M.—Slightly less than Alpha.

Mar. 1.—Cloudy.

2.—Equal to Alpha.

3.—Slightly less than Alpha, but brighter than Algol.

4.—Brightness slightly decreased. In full moonlight it seems yellower than before.

5.—Same.

6.—Cloudy.

7.—One magnitude less than Alpha.

8.—Equal to Gamma.

9.—Cloudy.

10.—Seen between clouds only.

11.—Scarcely as bright as Delta.

12.—Fully as bright as Delta.

14.—Same.

15.—Same.

16.—Cloudy.

18.—It seems dimmer than Delta but brighter than Kappa in the clear dark sky of San José. It is noticeably orange tinted.

19.—Dimmer than Kappa.

20.—Same. About equal to Nu.

21.—Cloudy.

During the days following some marked fluctuations were noticed, but not being recorded at once, the date could not be afterwards recalled.

Mar. 27.—Equal to Kappa.

30.—Equal to Nu which is fainter than Kappa.

31.—Same. Nearly full Moon.

The March observations were taken at 8 P. M., but the decrease of altitude made 7:45 a more desirable hour since.

April 1.—Cloudy.

2.—Same.

3.—It is about one magnitude dimmer than Nu, but is brighter than 32 Flamsteed's Catalogue classed as 5.1
4.—Clouds in the west.
5.—Cloudy.
6.—In opera glass equal to 36.
7.—Dimmer than 36, slightly.
8.—It is visible easily without magnifying power being a magnitude brighter than 32.
9.—It is a shade dimmer but is still brighter than 32.
10.—Dimmer than 32 or 36 and equal to 30 classed as 5.4.
11.—Cloudy.
12.—In opera glass through a light layer of cloud it is brighter than 32 but dimmer than Kappa.
13.—Same. Night not very clear.
14.—Slightly dimmer than 30.
15.—Cloudy.
16.—Still dimmer than 30.
17.—Cloudy.
18.—It is much brighter than 32 and resembles Psi Persei classed as 4.2.
19.—It has waned to the size of 30.
20.—Slightly dimmer than 30.
21.—Still dimmer than 30, but brighter than adjacent star of 6.5, magnitude.
22.—Not brighter than 6.5.
23.—As bright as Kappa and is of a deep orange hue.
24.—Cloudy.
25.—Much dimmer than 30 but a little brighter than adjacent star classed as 6.5.
26.—Same.
27.—Nearly as bright as Kappa.
28.—Cloudy.
29.—In moonlight, twilight and at a low altitude and it seems a little brighter than 6.5.
30.—Cloudy.

On account of the interference of buildings, comparisons were difficult in May but the Nova gradually brightened again and on the 7th inst. was of nearly 4th magnitude. The charts furnished to POPULAR ASTRONOMY by the Rev. Mr. Hagen were a valuable aid in the foregoing observations.

ROSE O'HALLORAN.

SAN FRANCISCO, May 10, 1901.

Observations of Nova Persei.—Sky was cloudy every night from April 9th to 13th inclusive.

April 14	5.4	About same as No. 25 of Hagen's Catalogue.
16	6.0	Same as No. 42. Hagen's list.
17	4.3	Plainly visible to the naked eye again, nearly as bright as κ Persei.
18	4.5?	Still visible to the naked eye but clouds interfered before an observation could be made with the telescope.

ALTA, Iowa,
April 19, 1901.

DAVID E. HADDEN.

Observations of Nova Persei at the University of Virginia.—The Nova was first seen independently by Mr. Micou at 7:30 P. M. Eastern time Feb. 22. The following are the estimates of magnitude obtained. The hours given are those of Mr. Micou's observations. Mr. Olivier's observations were made between 8 and 10 P. M. but the time is more specifically indicated in three cases: Feb. 23, 9^h; Feb. 24, $8^h\ 30^m$; Mar. 1, 10^h:

Date.	Time.	Micon.	Notes.	Olivier.	Notes.
	h m				
Feb. 22	7 30	0.5	bluish-white.	.....	
23	7	0.1		0.3	same as α Lyræ.
24	8	0.2		0.2	same as Capella
25	8	0.8	yellowish-white	0.7	
26	9	1.2		1.4	
27	8 & 10	1.8	reddish	1.8	
28	8	1.7		1.7	red
Mar. 1	8	2.2		2.3	red
2	8	2.4	dull red	2.15	
3	8	2.8		2.5	red
5	10	2.6		2.45	red
6	7 30	3.05		3.0	
7	7 30	3.05		2.85	
8	.....	.....		3.2±	through heavy smoke
9	8 30	3.6	partly cloudy	3.3±	very smoky
11	8	3.6			
12	8	3.3		3.3	
13	9	3.7	brilliant red	3.7	
14	9	3.4		3.3	
16	9	4.0		4.0	
17	8	3.8			
18	8	4.0		3.9	
19	8	5.0		5.0	
21	8	5.2		.....	red and has been since Mar. 1.
22	8	4.8			
24	8	5.2			
26	8	3.9		3.9	
28	9	5.3			
29				5.2	
31	8	5.4	full moon	4.6	
Apr. 7	8	5.6			
9	9	5.3			
10	8 20	5.6			
11	8	5.6			
16	8	5.6			

RICHARD D. MICOU AND CHARLES W. OLIVIER.

UNIVERSITY OF VIRGINIA.

Observations of Nova Persei.—The new star was discovered here independently on the evening of Feb. 24, when the sky cleared after three days of cloudy weather. The star was at that time brighter than the first magnitude, and of a reddish tint. The following observations are estimates, made in terms of grades, the mean value of one grade corresponding almost exactly to one-tenth of a magnitude. This value of the grade was used where none could be inferred directly from the observations. Up to and including April 9, the estimates were made with the unaided eye, after that time with the aid of field-glasses. The time is Greenwich mean time:

G. M. T. 1901.	Magn.	Estimates.
Feb. 24.58	1 +	Estimated magnitude.
25.58	1.0	" "
26.58	1.2	" "
27.58	1.7	" "
Mar. 1.66	2.5	" "
2.63	2.3	v 1 β
3.62	2.7	β 3 v
5.54	2.8	β 5 v, β to v = α to β, v . 6 ϵ to β, v . 5 δ to β
11.62	3.5	δ 4 v 3 ν
15.58	3.7	v 2 ν
17.58	3.7	v 2 ν
23.62	4.0	v = ν, κ = v
24.54	4.0	ν 1 v
31.62	4.4	v = σ, ι 1 v
April 4.58	4.0	v = ν and decidedly brighter than σ
8.58	4.5	μ 3 v, v = σ, κ 4 v
9.62	5.0	v 1 *l*, both plainly seen
18.56	4.6	v 4 *l*, v .3 *l* to ν, v = σ
19.62	5.7	*l* 4 *a* 4 v
20.58	6.0	*a* 5 v 5 *b*, *c* 5 v 5 b, *l* 3 *a* 5 v, v 4 *d*, *d* 2 *b*
25.58	5.8	*c* 5 v 5 *b*, *a* 2 v, v = *e*
26.58	6.0	*c* 8 v 5 *b*, *e* 1 v, *g* 4 v, v 5 *d*, *a* 4 v 5 *b*
28.60	5.7	*c* 4 v = *a*, v = *f*, *g* 3 v 2 *h*

REMARKS: The comparison stars *a*, *b*, *c*, *d*, *e*, *f*, *g*, are respectively, Nos. 25, 38, 37, 33, 46, 47, 45, of Professor Hagen's Catalogue published in the April number of POPULAR ASTRONOMY:

Feb. 26. Red.
Feb. 27. Yellow.
April 4. Perseus seen during a partial clearing of the sky.
9. Altitude 30°, sky somewhat hazy.
28. Bright Moon, altitude of Nova 15°, sky very clear.

Early in April, and again about the middle of the month, there seem to have been decided irregularities in the course of the star's decline. The observations are quite accordant, and indicate the reality of these fluctuations. In the field-glasses the star has appeared of a decidedly red tint, very similar to that so characteristic of long-period variables.

W. C. BRENKE.

UNIVERSITY OF ILLINOIS OBSERVATORY,
1901, May 10.

Nova Persei.—The following are my estimates of the brightness of Nova Persei upon the dates indicated:

1901.	Magnitude.	Comparisons.
March 2	2.3	α Persei 4 v
6	3.0	v 1 δ Persei
7	2.9	v 1 γ Persei
12	3.1	v = δ Persei
16	4.1	ν Persei = v = κ Persei
19	5.1	v = No. 28
21	4.5	v = σ Persei
22	4.5	v = σ Persei
27	4.2	κ Persei 1 v
29	4.3	κ Persei 1 v, v 2 σ Persei
April 1	4.3	κ Persei 1 v, v 2 σ Persei
5	4.3	κ Persei 2 v, v 2 σ Persei
12	4.3	κ Persei 2 v, v 2 σ Persei
17	5.0	v 1 No. 28
26	5.2	No. 25 1 v, v 1 No. 37

1901.	Magnitude.	Comparisons.
April 27	5.1	v 2 No. 37
May 1	5.4	v = No. 25
3	5.4	v = No. 25
6	5.4	v = No. 25

During the month of March comparisons were made with the naked eye; after April 1st, field glasses were used.

Where stars are designated by numbers, they are taken from Hagen's "Catalogue and Chart for Observing Nova Persei."

ANNE SEWELL YOUNG.

MT. HOLYOKE COLLEGE,
May 9th, 1901.

Ephemeris of Eros.—The accompanying ephemeris of Eros is for Berlin mean midnight. During July the planet will be so near the western horizon at nightfall that observations of it will probably not be made.

1901.	R. A.			Decl.			log Δ
	h	m	s	°	′	″	
June 1.5	10	27	12.3	— 7	52	13	9.964247
2.5	10	30	4.0	8	3	37	9.967995
3.5	10	32	55.3	8	14	59	9.971733
4.5	10	35	46.1	8	26	20	9.975463
5.5	10	38	36.6	8	37	39	9.979183
6.5	10	41	26.7	8	48	56	9.982895
7.5	10	44	16.4	9	0	12	9.986597
8.5	10	47	5.7	9	11	26	9.990291
9.5	10	49	54.7	9	22	38	9.993975
10.5	10	52	43.3	9	33	49	9.997649
11.5	10	55	31.5	9	44	58	0.001313
12.5	10	58	19.4	9	56	6	0.004968
13.5	11	1	7.0	10	7	12	0.008612
14.5	11	3	54.3	10	18	17	0.012246
15.5	11	6	41.3	10	29	20	0.015870
16.5	11	9	28.0	10	40	22	0.019482
17.5	11	12	14.4	10	51	23	0.023084
18.5	11	15	0.4	11	2	22	0.026674
19.5	11	17	46 1	11	13	19	0.030252
20.5	11	20	31.5	11	24	15	0.033817
21.5	11	24	16.7	11	35	9	0.037371
22.5	11	26	1.5	11	46	2	0.040913
23.5	11	28	46.0	11	56	53	0.044442
24.5	11	31	30.3	12	7	42	0.047958
25.5	11	34	14.2	12	18	29	0.051462
26.5	11	36	57.9	12	29	15	0.054953
27.5	11	39	41.3	12	39	58	0.058430
28.5	11	42	24.4	12	50	40	0.061896
29.5	11	45	7.2	13	1	20	0.065347
30.5	11	47	49.7	— 13	11	58	0.068786

MARY CLARK TRAYLOR.

UNIVERSITY OF DENVER, May 10, 1901.

Variation in Light of Eros.—The range of variation in the light of Eros, which has been diminishing during the spring has now become zero. In February, 1901, it was found by European astronomers to amount to 2.0 magn. Observations by Professor O. C. Wendell, with the Harvard equatorial, showed that the range on March 12, 1901, was 1.13 magn., on April 12, it was 0.40 magn. and on May 6 and 7 it was imperceptible and apparently less than 0.1 magn.

EDWARD C. PICKERING.

HARVARD COLLEGE OBSERVATORY, Cambridge, Mass.
May 8, 1901.

An Introduction to Stellar Astronomy is the title to a book recently prepared by W. H. S. Monck of Dublin, Ireland. The greater part of this book, by favor of its author has, from time to time, appeared in this publication, and our readers have profited by it, because the various articles have contained useful data gathered by a busy but interested writer in the midst of exacting professional duties.

It is a matter of gratification to us, and doubtless to all interested in stellar astronomy to have these separate articles and other appropriate matter collected and arranged in book form, for the more ready reference pertaining to a theme that is growing so rapidly that it will soon require the knowledge of a specialist to use its increasing data effectively.

As arranged in chapters this book first gives a brief historical statement pertaining to the nature and distances of the fixed stars; then, of their magnitudes and brightness, as determined by photometry, their distribution in space and a comparison of their light with that of sunlight. The third chapter treats of the distances of the stars which always interests everybody who gives but passing thought to such a grand theme. It is so described that most readers will easily understand how the knowledge of star distances is obtained, and also get some definite idea of the great problem of the extent of the sidereal universe. Chapter 4 deals with the motions of the Sun and the stars; 5, the spectra of stars and their causes; 6, the discoveries made by means of the spectroscope; 7, double stars and star systems; 8, variable stars, new stars and nebulæ; 9, comets and meteors, followed by a list of 253 bright stars in table form with place, magnitude, spectrum and remarks. This very useful book contains 203 pages and is published by Messrs. Hutchinson & Co., Paternoster Row, London, England. Price 3 shillings 6 pence.

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Lantern Slides at Goodsell Observatory.

LIST OF NEGATIVES FROM WHICH GOOD LANTERN SLIDES CAN BE MADE.

THE SUN.

A. 1. Aug. 4, 1893 } Sections from seven photographs showing development of a great sun-spot group.
2. " 7, " }
3. " 11-11, " }
6. Enlarged view of sunspot Aug. 7, 1893, direct enlargement in telescope.
7. Enlarged view of sunspot, Aug. 16, 1894.
8. " " " " " 17, 1894.

THE MOON.

B. 1. Crescent, June 6, 1894.
2. Crescent, July 19, 1893.
3. First Quarter, July 20, 1893.
4. Gibbous, July 21, 1893.
5. Full, Dec. 13, 1894.
6. Gibbous, Oct. 19, 1894.
7. Enlargement direct in telescope showing S. W. quadrant, March 27, 1893.
8. Enlargement showing N. W. quadrant, March 27, 1893. Scale: whole diameter of Moon equals 7 inches.
9. Enlargement Oct. 18, 1894.

NEBULÆ AND STAR CLUSTERS WITH 8-INCH EQUATORIAL.

			Exposure.
C.	1.	Orion nebula.....	10m
	2.	" "	2h
	3.	" "	4h 40m
	4.	" "	9h
	5.	Great Spiral Nebula.....	3h
	6.	Andromeda Nebula.....	12h
	7.	Cluster Scutum Sobieskii.....	8h
	8.	Double Cluster in Perseus.....	1h 20m
	9.	Pleiades	4h
	10.	Ring Nebula in Lyra	2h

COMET AND CONSTELLATION PICTURES WITH 2½-INCH CAMERA.

			Exposure.
D.	1.	Swift's Comet, April 28, 1892....	55m
	2.	Rordame's Comet, July 11. 1893..	43m
	3.	Rordame's Comet, July 18, 1893..	60m
	4.	Scutum Sobieskii region.....	8h
	5.	Andromeda Neb. region.....	3h 30m
	6.	Trails of Polar stars.....	2h 30m
	7.	Gamma Cygni region...	4h
	8.	Pleiades region...	4h
	9.	Orion region.....	4h 30m
	10.	Lyra, region of Ring Nebula....	2h
	11.	Scorpio..	4h

MISCELLANEOUS.

E. 1. Goodsell Observatory.
2. Lightning } Two fine specimens taken in
3. Lightning } Aug., 1889.

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